JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS
DECEMBER 1975

RISK AND THE RATE OF RETURN ON FINANCIAL ASSETS: SOME OLD WINE IN NEW BOTTLES

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Strides have been made recently in the discovery and refinement of theoretical models which purport to describe the relationship between asset prices and their risk attributes. (See especially Lintner [13,14,15], Sharpe [19], Mosin [17,18] and Fama [7,8.9].) The models have gained widespread acceptance because of their intuitive appeal and because most reported empirical evidence [1,4,5,11,20,21] allegedly supports their predictive value. It is our purpose to analyze critically one aspect of the nature of this evidence, reveal its inherent weakness, and to design an alternative test to examine the risk-return function. After observing the performance of an extremely large number of issues over long periods of time, we find little support for the notion that risk premiums have, in fact, manifested themselves in realized rates of return.

I. The Bull-Bear Market Problem

Timing is critical in any test of the relationship between risk and expected rates of return. If the assumption that investors' expectations are borne out on average is violated, a systematic error crops up which tends to bias the results in one direction or the other.

To prove this we need only assume, in the convention of Sharpe, Lintner, and Fama, that the underlying distributions generating the rates of return to financial assets over any period t are bivariate normal, and thus, are linearly related to the rate of return to the market portfolio, p:

$$r_{j,t} = a_j + \beta_j p_t + e_{j,t}$$

where we assume that $a_j = E(r_f)\ (1-\beta_j)$ with $E(r_f)$ equal to the expected rate of return accruing to an issue devoid of systematic risk.

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An expanded treatment of this topic is provided by the authors under the title of "On the Evidence Supporting Risk Premiums in the Capital Market" and is available as Wisconsin Working Paper 4-75-20 from the Graduate School of Business, University of Wisconsin, Madison.

Under these assumptions if we sample from the *stationary* distributions over a number of periods, t = 1, 2,..., n, each of which is equal to the investors' horizon, our *sampling* mean can be expressed by:[1]

$$\bar{r}_j = 1/n \sum_{t=1}^{n} [a_j+\beta_j p_t+e_{jt}] = a_j + \beta_j\bar{p} + \bar{e}_j. \tag{1}$$

If we assume that the sample covariance between e_{jt} and p_t is zero, by substituting (1) into the definitional form of the *cross-section* covariance between β_j and $\bar{r}_j$ we get:[2]

$$\mathrm{Cov}_{\beta_j,\bar{r}_j} = \bar{p}\sigma^2_{\beta_j} + \mathrm{Cov}_{a_j,\beta_j} + \mathrm{Cov}_{\bar{e}_j,\beta_j},$$

where under our assumptions it is true that:

$$\mathrm{Cov}_{a_j\beta_j} = -E(r_f)\sigma^2_{\beta_j}.$$

Thus, the slope coefficient of the cross-section regression of risk on return can be written as

$$b_\beta = \frac{\mathrm{Cov}_{\hat{\beta}_j,\bar{r}_j}}{\sigma^2_{\hat{\beta}_j}} \{E(p) - E(r_f)\} + \{\bar{p} - E(p)\} + \frac{\mathrm{Cov}_{\bar{e}_j,\beta_j}}{\sigma^2_{\beta_j}}$$

where E(p) is the *expected* rate of return to the market portfolio.

Since we can safely assume that the final term of (3) is small for a

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[1]For a discussion of the problem of misspecification of the horizon, see Jensen [11, p. 186].

[2]If the risk-free rate, R_f, is not constant over the sampling period, one is likely to obtain a biased estimate of β_j with (2) unless the risk-free rate is subtracted in each year from both the independent and the dependent variables. Our results show, however, that the bias is likely to be small.

representative cross section of well-diversified portfolios all highly correlated with the market, the following approximation for testing across these portfolios should hold up well:

$$b_\beta \approx [E(p) - E(r_f)] + [\bar{p} - E(p)]. \tag{4}$$

The first term in brackets can be taken to be the slope of the true risk-return function; the second term is the expectation error. In a bear market, where, on average, results exceed expectations, we *underestimate* the slope of the true function; in a bull market we *overestimate*. Thus, we are faced with the unfortunate situation that the nature of our empirical result is determined by the nature of the market we sample in.

The single means of attacking the problem is to assume stationarity in the underlying probability distributions over long periods of time and to sample over these intervals, hoping to obtain an accurate picture of the distributions by increasing the sample size. In light of this we would suggest that the timing of the empirical tests conducted thus far has been unfortunate since they have sampled a relatively small number of observations within the bullish market, 1953 through 1968.[3] It may prove interesting to sample over other long run periods when the possibility that stock returns have generally exceeded expectations is less distinct. We shall direct our attention to the design of such a test in Part III.

II. Continuing the Search for Risk Premiums

A. Design of the Test

We construct sample portfolios from stocks selected from those listed on the New York Stock Exchange in 1926.[4] No attempt is made to prescreen the stocks to assure their survival over the period observed.[5] Each portfolio consists of 25 stocks, and 114 such portfolios are constructed.[6] Monthly

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[3]During this period the geometric average rate of return to all stocks listed on the New York Stock Exchange was 15.09 percent.

[4]The tapes which were originally developed by the Center for Research on Security prices (CRISP) at the University of Chicago, were revised and updated by the Standard and Poor's Company. The source of all our data is the CRISP tapes.

[5]A comparison of the breadth of the risk spectrum exhibited by our portfolios and that exhibited by portfolios held by mutual funds, reveals little discernible difference.

[6]The number of portfolios is determined by constraints on computer time. In measuring the rates of return to all the portfolios, the compounding interval is continuous.

performance relatives are calculated for each portfolio from February 1926 to December 1971 by taking the arithmetic mean of the performance relatives for the 25 stocks in each portfolio. This is tantamount to assuming that a given number of dollars is divided equally among each of the 25 stocks in a portfolio at the beginning of a month and held until the end of the month, at which time the value of the portfolio (with distributions and adjustments) determines the performance relative for the portfolio. If a stock in any portfolio is delisted from the Exchange, for any reason, a new stock is selected to take its place in the portfolio at the time of delisting.

From the monthly performance relatives for the 114 portfolios, we calculate the geometric mean of the monthly rates of return and standard deviation of the monthly returns over the entire 46-year period and nine shorter periods of five years. (The last period is six years, to be precise.) The period 1946-1971 is of special interest because it is a period over which the variance of the monthly performance relatives is relatively constant.

We also make separate calculations of the β coefficient for each portfolio (when regressed on the average return for all stocks on the CRISP tapes), in addition to the coefficients of skewness and kurtosis. Since the portfolios are well diversified, residual variance can be taken to be small relative to the portfolios' own variance.[7]

B. What to Expect

Before proceeding to the results, we should take a moment to reconsider what one might expect in light of the alternative theories available. If risk premiums are nonexistent, the long periods should show little or no relationship between portfolio returns and portfolio standard deviations (or between β coefficients and portfolio returns). That is, if:

$$E(r_i) = E(r_j) = E(p) = E(r_f), \text{ for all } i \text{ and } j$$

and

$$\bar{p} = E(p)$$

[7]This is confirmed by the fact that the product moment correlation coefficient between the rates of return to the portfolios and the average return for all stocks falls within the range .90 to 1.00 over all the 114 portfolios. In every case we correlate the rates of return as opposed to the risk premiums ($r_{jt}-r_f$ and p_t-r_f; where r_f is the yield to maturity on a one-period risk-free bond at the beginning of period t). While an improved estimate of the β coefficient can be obtained by relating the risk premiums as opposed to the total rates of return, Miller and Scholes [16] have shown that the difference between the estimates is small. Our analysis confirms this finding.

then

$$\text{Cov } \bar{r}_j, \hat{\beta}_j = 0.$$

This merely states that if the sampling period is one over which the sample mean of market returns is close to expectations, then there should be no correlation between observed portfolio rates of return and observed variances or β coefficients.

For shorter periods, if $\bar{p} \neq E(p)$, we expect to observe that:

$$\text{when } \bar{p} > E(p), \text{ Cov } \bar{r}_j, \hat{\beta}_j > 0$$

$$\text{when } \bar{p} < E(p), \text{ Cov } \bar{r}_j, \hat{\beta}_j < 0.$$

That is, during bull markets (results better than expectations), we expect to observe that portfolio rates of return are positively related to portfolio variances, and vice-versa during bear markets.

C. The Results

Our principal results are shown in Tables 1 and 2. Table 1 shows the regression relationships between sample means and standard deviations and β coefficients for two long periods 1926-71 and 1946-71. Given expression (1), the results of Table 1 would seem to imply that *if* the true risk return function were positively sloped over the period 1946-71, the rates of return which in fact materialized in the period failed to meet expectations.[8]

The effects of the *time-period* problem are apparent when looking at the results for the five-year periods. Table 2 shows the results of regressions of returns on standard deviations for each of the periods. In addition, in each case we compare the average performance in the market with market performance over the ten years preceding the period in question. The latter comparison is significant because it may provide some notion of how market performance in any period compared with expectations based on past performance. Note that in all cases, when the market performance during a period exceeded the previous ten-year performance (indicated by a + sign in the last column),

[8] Because of the attractiveness of the period of relatively stable variance, 1946-71, we might note a few more statistics for this period. The arithmetic mean of portfolio returns is also negatively related to the standard deviation (T-ratio = -2.97). And, a multiple regression of the geometric mean on the standard deviation, skewness and kurtosis yields the following (T-ratios in parentheses):

$$\bar{r} - .015 - .098\sigma_\lambda - .039\text{SKEW} - .004\text{KUR}$$
$$(11.33)\quad (-6.01)\quad (-0.48)\quad (-1.43)$$

TABLE 1

REGRESSION STATISTICS FOR LONG PERIODS

Statistics	1926-71	1946-71
	Geometric means regressed on standard deviations	
b_σ coefficient	-.0353	-.0945
T-ratio	-4.87	-5.82
	Geometric means regressed on β coefficients	
b_β coefficient	-.0031	-.0043
T-ratio	-4.79	-5.42
Average of portfolio returns	.0094	.0089
Average of Standard Deviations	.0893	.0460

TABLE 2

COMPARISON OF SIMPLE REGRESSION OF PORTFOLIO RATES OF RETURN ON PORTFOLIO VARIANCES FOR FIVE-YEAR PERIODS WITH CHANGES IN MARKET PERFORMANCE

Period	Regressions b_σ-coef	T-ratio	Average* Market Rate	Change from Last Ten Years
1926-30	-.035	-0.73	-.0048	-.0003**
1931-35	-.045	-1.70	.0173	-.0063**
1936-40	-.098	-5.58	.0012	-.0049
1941-45	+.212	+7.12	.0258	+.0166
1946-50	-.162	-4.08	.0063	-.0071
1951-55	-.234	-4.46	.0142	-.0018
1956-60	-.101	-2.55	.0085	-.0023
1961-65	+.292	+6.35	.0119	+.0006
1965-71***	-.041	-1.33	.0072	-.0040

*Geometric mean of monthly performance with all stocks on the CRISP tapes.

**Change from previous ten years using the continuously compounded monthly average rate of growth in the Standard and Poor's Index of 425 industrial stocks for both the earlier and later periods.

***Six-year period.

the relationship between returns and standard deviations is positive. And, when market performance was less than the previous period, the relationship between returns and standard deviations is negative. These results coincide strongly with our bull-bear market hypothesis.

III. Summary and Conclusions

Our purpose has been twofold. First, we point out a conceptual shortcoming in previous empirical efforts that generally support the concept of a risk premium and measure the tradeoff between risk and return. Uncontained, the problem outlined in Part I of this paper is capable of generating the inference that premiums are awarded for risk taking in the stock market when *in fact* no risk premiums exist.

Second, we empirically measure the risk return relationship over various time periods to reveal the severity of the bull-bear market problem. The results of our empirical effort do not support the conventional hypothesis that risk--systematic or otherwise--generates a special reward. Indeed, our results indicate that, over the long run, stock portfolios with lesser variance in monthly returns have experienced greater average returns than their "riskier" counterparts.

The implications of all this are not abundantly clear. But, it would seem that the search for new theories of relative asset pricing in the face of the differential attributes characterized as "risk" is not over.

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