

The efficient market inefficiency of capitalization-weighted stock portfolios

"Matching the market is an inefficient investment strategy."

Robert A. Haugen and Nardin L. Baker

As of the end of 1990, the largest 200 defined-benefit pension plans had indexed a combined total of approximately \$200 billion to capitalization-weighted stock portfolios, such as the Wilshire 5000. The sponsors of these plans presumably believe this to be an efficient investment, in the sense that alternative portfolios with the same expected return but significantly lower volatility aren't readily available.

The purpose of this article is to show that cap-weighted stock portfolios are *inefficient* investments. We shall argue moreover that, except under extremely restrictive conditions, theory predicts them to be inefficient, even if the capital market is assumed to be informationally efficient, and even if all investors rationally optimize the relationship between risk and expected return in equilibrium.

A prediction that cap-weighted stock portfolios have the lowest possible volatility given their expected return must be based on a set of assumptions that includes the following:

1. All investors agree about the risk and expected return for *all* securities.
2. All investors can short-sell *all* securities without restriction.
3. No investor's return is exposed to federal or state income tax liability now in effect.
4. The investment opportunity set for *all* investors holding any security in the index is restricted to the securities in the cap-weighted index.

Absent these assumptions, even the most com-

prehensive cap-weighted portfolios occupy positions *inside* the efficient set. In this context, theory directs us to manage risk in seeking out investment portfolios with the lowest possible volatility given expected return.

WHEN CAP-WEIGHTED PORTFOLIOS ARE EFFICIENT

The Wilshire 5000 is the most comprehensive cap-weighted index of the U.S. equity population. It also represents the pooled holdings of the member stocks for all investors who have taken positions in any of the stocks in the index. Thus if, for every investor owning any of the stocks in the index, we separate these holdings and then aggregate, the combined portfolio will be the Wilshire 5000.

Under what conditions will the combined portfolio be the efficient set relative to the U.S. equity population?

Suppose all who hold stocks in the index invest only in U.S. equities with no restrictions on their ability to sell short. Also assume that all investors agree about the covariance and expected return characteristics for all stocks. In this case, they all face the same unconstrained efficient set, such as the one depicted in Figure 1A. (The figure is drawn for an assumed three-stock population for explanatory purposes.) Investors take portfolio positions on their unconstrained efficient frontier. (These positions might be represented by points X, Y, and Z in the unconstrained efficient set of Figure 1A.)

With no limitations on short-selling, when you combine two or more of these efficient portfolios, you obtain a combined portfolio that is also efficient. This property is illustrated in Figure 1A and Figure 1B (which plots percentage of assets invested in one stock against another).

The percentage invested in stock A is plotted on the horizontal axis of Figure 1B; the percentage in B is plotted vertically, and the percentage in C is implied. The three points of the triangle in Figure 1B represent exclusive investments in each of the three stocks, respectively. That is, the origin of the graph represents a 0% investment in A and B and a 100% investment in C. Positions outside the triangle require short-selling at least one of the three stocks.

With no restrictions on selling short, you can take any position you want in Figure 1B. Each of the portfolios represented in Figure 1B falls on or within the unconstrained efficient frontier of Figure 1A. The combinations of stocks A, B, and C represented on the unconstrained efficient frontier all fall on the straight line of Figure 1B. Markowitz [1952] calls this line the critical line, and it must always be linear if short-selling is unrestricted.

Now consider the portfolios labeled X and Y in the two figures. If we combine these, we might get Z, which is always on the unconstrained critical line of Figure 1B, and also on the unconstrained efficient frontier of Figure 1A.

When short-selling is unrestricted, combinations of efficient portfolios are themselves efficient. As the Wilshire 5000 is, under these conditions, a combination of the unconstrained, efficient portfolios held by all investors, it also will be positioned on the unconstrained efficient frontier. Because we have assumed that everyone sees the same efficient fron-

tier, the 5000 will be perceived as an efficient portfolio by all.

WHEN CAP-WEIGHTED PORTFOLIOS ARE INEFFICIENT

What If We Disagree?

Suppose that investors have differing opinions about the risks and expected returns of stocks. Fama [1976] has shown that, in the context of unrestricted short-selling and the existence of a risk-free asset, in equilibrium the cap-weighted market index will be efficient when viewed in the context of the weighted averages of all investors' expectations about stock returns and covariances. In taking the weighted average, we weight investors' expectations by their marginal rate of substitution of expected future consumption for variance of future consumption.

While the cap-weighted market index is efficient in this sense, no one investor sees it as efficient except under the highly unlikely circumstance that her views are the weighted averages across all stocks. Moreover, no investor invests in the cap-weighted index; rather all manage risk in the context of their own expectations and take an optimal position on their own perceived efficient frontier.

Some might suggest that those who invest in cap-weighted portfolios believe the market to be informationally efficient. Because of this they are willing to accept the mean-variance efficiency of the cap-weighted index based on consensus market expectations. In an informationally efficient market, however, security prices reflect the views of *fully-informed* investors as opposed to the consensus views of *all* investors.

FIGURE 1A

CONSTRAINED AND UNCONSTRAINED EFFICIENT SETS

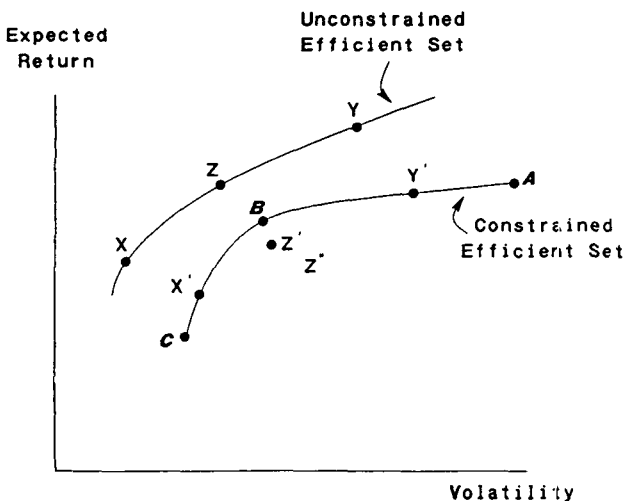
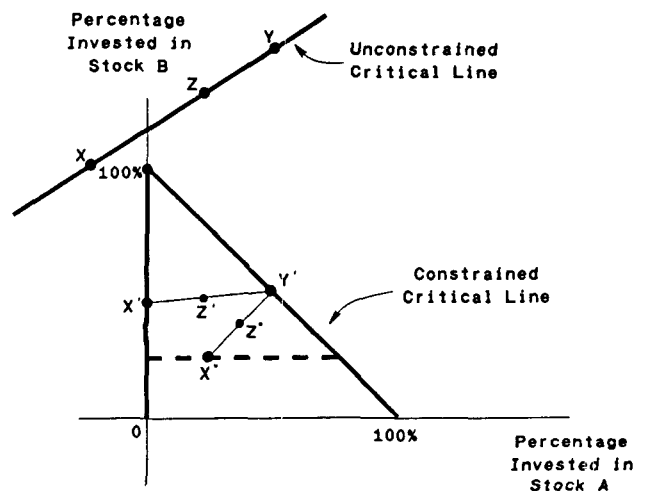


FIGURE 1B

CONSTRAINED AND UNCONSTRAINED CRITICAL LINES



What If Some Of Us Can't Sell Short?

Now let's strike the assumption that everyone can invest the proceeds of short-selling without restriction. In fact, in the real world, very few of us have this opportunity. With restrictions on short-selling, we lose the convenient property that combinations of efficient portfolios are themselves efficient.

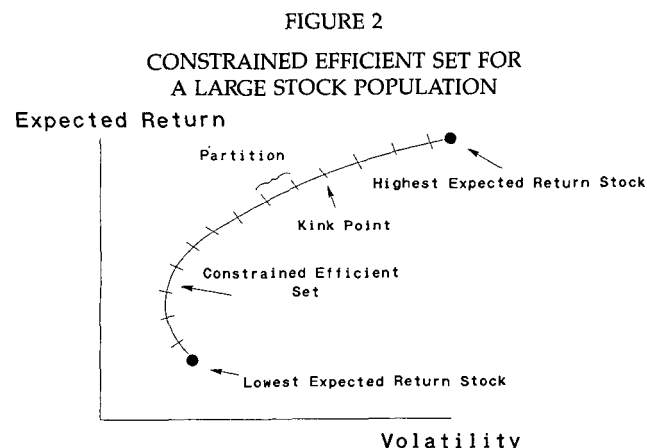
To see this once again, consider Figure 1B. If we can't sell short, we must take positions on, or within, the borders of the triangle. For this example the undominated positions closest to the unconstrained positions on the critical line fall along the vertical axis and along the triangle's hypotenuse. These positions are represented on the constrained efficient set of Figure 1A.

Now suppose we combine X' with Y' . We get Z' , which is *inside* both the triangle of Figure 1B and the efficient set of Figure 1A. When we can't sell short, combinations of efficient portfolios are no longer themselves efficient.

In a more general context, with large populations of securities, the constrained efficient set looks as it does in Figure 2. The minimum volatility set is bounded on each end by the highest and lowest expected return stocks. As you move along the set toward the center from each end, you move past various "kink points," which serve to partition the set.

Within each partition, a given set of two or more stocks are positively weighted for all portfolios within the partition. As you cross a kink, one of the stocks drops out of the set, or perhaps an additional stock is added. While combinations of two or more portfolios taken from a single partition are themselves efficient, combinations of portfolios taken from different partitions will be inefficient.

The constrained efficient set for the market as a whole includes many partitions. This means that, when we combine the efficient portfolios held by investors (many of whom cannot sell short without



restriction), the aggregate market index is no longer efficient.

Tax Avoidance

The existence of an income tax on investment returns further drives the cap-weighted market index to a position of relative inefficiency. Investors differ in their exposure to federal taxes. Dividends received by corporations, for example, have been sheltered by the dividend exclusion; investors in different brackets are taxed at different rates; investors with different trading histories will have different cost bases for calculating gains and losses, and the tax laws of the states, municipalities and foreign countries differ widely. Thus, there will be distinct differences in the after-tax opportunity set facing each investor. A portfolio that is after-tax efficient for one investor will be inefficient for another, and the cap-weighted aggregate of these portfolios is unlikely to be efficient for a pension fund that pays no taxes.

To see this, once again consider Figure 1B. Suppose we have a tax-exposed investor with accrued capital gains in stock C. She may have a critical line that includes positive positions in this stock. This after-tax critical line is represented by the broken line of Figure 1B with the tax-exposed investor taking a position at X'' . If this position is combined with Y' , we get an even more inefficient aggregate, positioned at Z'' in Figures 1A and 1B.

Human Capital

The existence of alternative investments that are not (and, in many cases, cannot be) included in the market index also contributes to its inefficiency. One factor important to the vast majority of investors is their human capital (the present value of future earned income). Rational investors will take positions that are efficient with respect to their human capital. Moreover, the risk-related properties of their investments in human capital will differ markedly according to training and occupation.

Once again, an equity portfolio that makes sense to one investor will appear inefficient to another when viewed out of context. Why should a cap-weighted combination of these financial investments be an efficient investment from anyone's perspective? Cap-weighted indexes are unlikely to be efficient investments for a particular pension plan, given its peculiar set of constraints and the nature of the business of its sponsoring firm.

Foreign Investors

Foreign investors play an increasingly important role in the U.S. capital market. If foreigners are

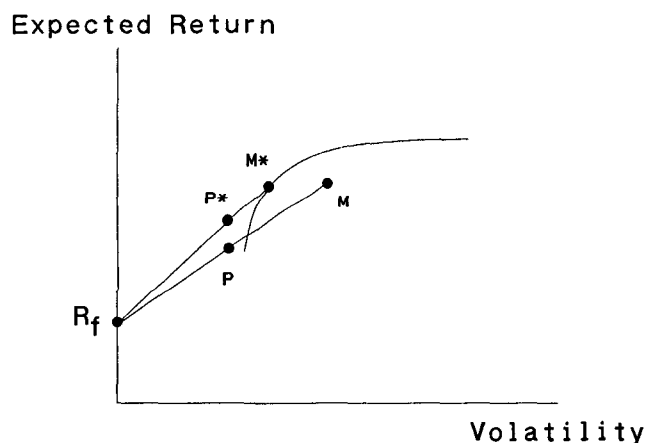
investing rationally, they will not build portfolios of U.S. securities that are efficient relative to the U.S. opportunity set. Rather, they will invest in securities that combine well with their domestic investments and their investments in other countries. Nevertheless, their holdings of U.S. securities are a significant part of the pooling process in building to the Wilshire 5000 index, again contributing to its inefficiency relative to the U.S. opportunity set.

THE BENEFITS OF EFFICIENT INVESTING

In these more realistic contexts, if pension funds attempt to match the market, they must bear uncompensated risk. This means that alternative portfolios are available with lower volatility and the same expected return as the cap-weighted indexes. Financial theory directs us to manage risk and invest efficiently. Equity managers should be assessed relative to an efficient benchmark, such as M^* in Figure 3, with low volatility relative to the inefficient market index denoted by M .

FIGURE 3

THE BENEFIT OF EFFICIENT INVESTING



The potential benefits of investing efficiently are substantial. Pension funds typically combine domestic equity investments with fixed-income investments in a sixty/forty split. If fixed-income investments are taken to be riskless, they might occupy a position at R_f in Figure 3. By combining M with risk-free bonds in a sixty/forty split, we get to P , but by combining M^* with the bonds in a seventy/thirty split, we get to P^* . The gap in expected return between P and P^* represents the benefit of efficient investing in the equity market.

AN EMPIRICAL TEST OF THE EFFICIENCY OF THE CAP-WEIGHTED INDEX

The most comprehensive cap-weighted index is the Wilshire 5000 index. To determine its relative efficiency, we shall attempt to build a low-volatility

stock portfolio and then examine its (out-of-sample) risk and return characteristics relative to the cap-weighted index and two sets of randomized portfolios.

Data and Methodology

The low-volatility portfolio is constructed from a population of 1000 U.S. stocks that are the largest in market capitalization over all U.S. exchanges and markets at the beginning of each of the years covered by the simulation experiment. The population is free from survival bias because we select the sample from all issues in existence at the beginning of each year, including issues that subsequently go bankrupt or are acquired by other firms.

The simulation experiment spans sixty-eight quarters from 1972 through 1989. At the beginning of the first quarter of 1972, we compute the weights in the lowest-volatility portfolio of the 1000 stocks in the preceding twenty-four months, subject to constraints that insure diversification and, hence, predictive power out-of-sample:

1. No more than 1.5% of the portfolio can be invested in any one stock.
2. No more than 15% of the portfolio can be invested in any one industry.
3. The minimum weight in any given stock is zero.

Portfolio weights for the 1000 stocks are found that would have minimized volatility over the preceding twenty-four months. In any given quarter, the vast majority of the stocks have zero weights in the efficient portfolio. The number of stocks in the efficient portfolio typically ranges between 100 and 150.

To find the portfolio we use the complete covariance matrix of returns. We employ an iterative procedure (Von Holthenbalken [1975]), which provides an exact solution without inverting the matrix. Given the constraints imposed on the problem, unique solutions do exist, and, as we shall see, these solutions have a great deal of predictive power, in that the solution portfolios subsequently turn out to have relatively low volatility in the following three months.

The efficient, low-volatility portfolio is then held for the following quarter. At this point, a new efficient portfolio is computed again over the trailing twenty-four months, and positions are shifted from the old to the new optimally (so as to control annual turnover to approximately 30%). Transaction costs are assumed to be 2% round-trip throughout the analysis. The optimizing process is repeated at the beginning of each quarter through 1989.

As a basis of comparison, we construct two

sets of randomized portfolios, one with weighting structures identical to that of the efficient portfolio and a second set with a weighting structure similar to that of the Wilshire 5000.

The randomized portfolios in the first set are identical to the efficient portfolio, in terms of the structure of their weights at any given point in time and with respect to the turnover of stocks within each portfolio. The difference between the portfolios in the first set and the efficient portfolio is that the identity of the stocks that fill the weights is randomly selected from the 1000 stock universe. In this sense the scatter provides an excellent basis for determining the probability that the risk attributes of the efficient portfolio have been achieved by chance. Indeed, the efficient portfolio is a member of the complete set of randomized portfolios that can be selected by chance.

The second set of 1000 randomized portfolios is cap-weighted. The number of stocks in these portfolios is equal to the average number in the efficient portfolio, and the stocks are randomly selected from the 1000-stock population. These portfolios should serve as a benchmark to assess the relative efficiency of the Wilshire 5000.

Results

The out-of-sample performance of the low-volatility portfolio is shown in Figure 4. Geometric mean return is plotted against annualized quarterly volatility. The darker scatter toward the upper right of the figure shows the comparative performance of 1000 randomized portfolios (not value-weighted). Obviously, the figure tells us that there is a very low probability that the efficient portfolio's low volatility can be attributed to chance. One would have to sample exhaustively before reaching a randomized portfolio with volatility characteristics that come close to those of the efficient portfolio. The distribution appears to be bivariate normal with the low-volatility portfolio 8.0 standard deviations away from the mean value for the 1000 randomized portfolios.

Note that the Wilshire 5000 index lies at the center of the lighter scatter of cap-weighted, randomized portfolios toward the lower left of the figure. The relative position of the efficient portfolio in mean-variance space indicates that both the Wilshire 5000 and both sets of randomized portfolios lie *within* the efficient set with a high degree of probability.

The difference in expected return for the two scatters undoubtedly reflects the size effect, which favored smaller firms during the period overall. The average market capitalization is smaller for the darker

FIGURE 4
RETURN AND VOLATILITY: 1972-1989

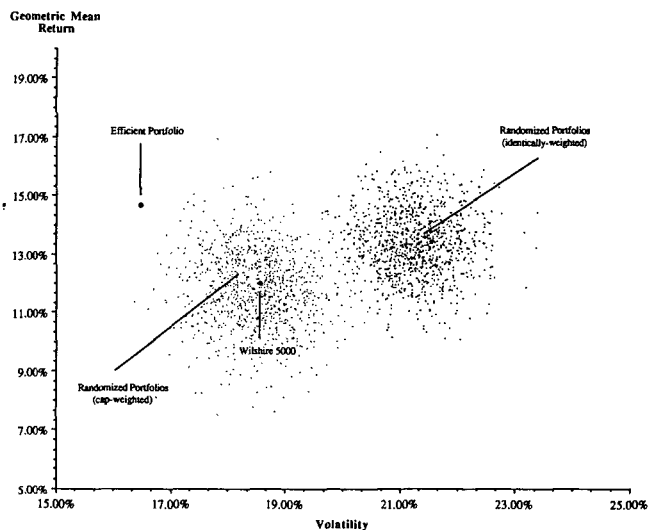
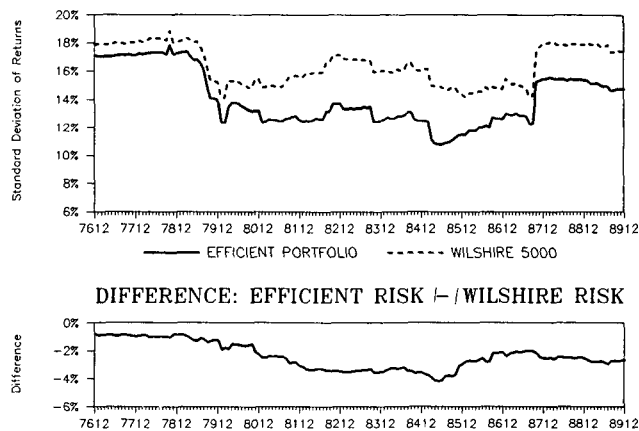


FIGURE 5
RISK COMPARISON
ROLLING FIVE-YEAR PERIODS

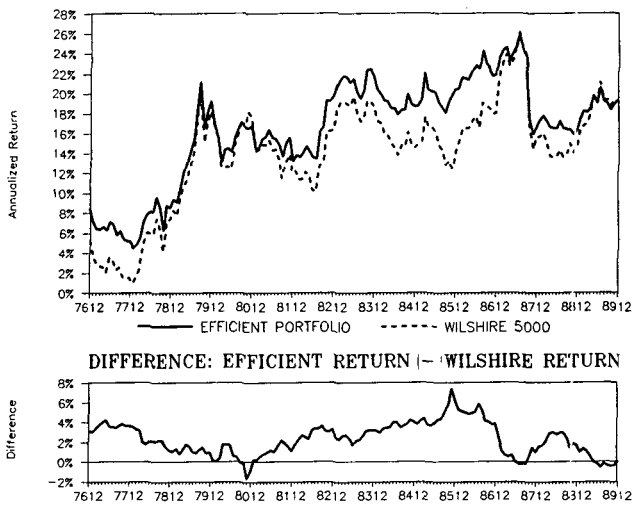


scatter because the weights for its portfolios are more nearly equal. The difference in volatility between the two scatters also reflects the smaller average capitalization for the darker scatter and the fact that smaller firms tend to have greater volatility.

To determine whether the dominance of the efficient portfolio is period-dependent, we plot the moving five-year volatility for the efficient portfolio and for the Wilshire 5000 in Figure 5. Again, the results are for the linked quarterly returns earned subsequent to establishing each quarter's portfolio weights. Note that the efficient portfolio consistently has lower volatility than the Wilshire 5000.

Figure 6 plots the five-year moving averages for the realized return. Note that, with brief exceptions, the efficient portfolio has consistently higher returns than the Wilshire 5000.

FIGURE 6
RETURN COMPARISON
ROLLING FIVE-YEAR PERIODS



The evidence supports the hypothesis that investment opportunities existed over the period 1972 through 1989 to build equity portfolios with equal or greater return but significantly lower volatility than cap-weighted portfolios in general and than the most comprehensive capitalization-weighted portfolio in particular — a result that is fully consistent with established theoretical arguments.

SUMMARY

Market-matching to domestic cap-weighted stock indexes is likely to be a suboptimal investment

strategy when investors disagree about risk and expected return, when short-selling is restricted, when investment income is taxed, when some investment alternatives are not included in the target index, or when foreign investors are in the domestic capital market. In the presence of these factors, there will be alternatives to cap-weighted portfolios that have the same expected return but lower volatility. This will be true even in the context of an efficient market where all investors take efficient mean-variance positions within the context of their individual tax exposure and within the context of the constraints placed on their portfolio weights, including the required investment in their human capital.

By the end of 1990, more than \$200 billion had been matched to cap-weighted indexes by the largest defined-benefit pension funds. We have provided theoretical arguments and empirical evidence here that matching the market is an *inefficient* investment strategy — even in an informationally *efficient* market.

REFERENCES

- Fama, E. F. *Foundations of Finance*. New York: Basic Books, 1976, p. 319.
- Markowitz, H. M. "Portfolio Selection." *Journal of Finance*, March 1952, p.77.
- Von Holthenbalken, B. "A Finite Algorithm to Maximize Certain Pseudo Concave Functions on Polytopes." *Mathematical Programming* 8, 1975.