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RISK, UNCERTAINTY, AND DIVERGENCE OF OPINION

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THE THEORY OF investor behavior in a world of uncertainty has been set out by several writers including Sharpe (1964) and Lintner (Feb. 1965). A key assumption of the now standard capital asset model is what Sharpe calls homothetic expectations. All investors are assumed to have identical estimates of the expected return and probability distribution of return from all securities. However, it is implausible to assume that although the future is very uncertain, and forecasts are very difficult to make, that somehow everyone makes identical estimates of the return and risk from every security. In practice, the very concept of uncertainty implies that reasonable men may differ in their forecasts.

This paper will explore some of the implications of a market with restricted short selling in which investors have differing estimates of the returns from investing in a risky security.[1] Explanations will be offered for the very low returns on the stocks in the highest risk classes, the poor long run results on new issues of stocks, the presence of discounts from net value for closed end investment companies, and the lower than predicted rates of return for stocks with high systematic risk.

THE ARGUMENT

Let us start with a relatively simple financial market in which securities are risk free one year government bonds. Now let us assume a single common stock company is organized to carry out a one year project such as sending a trading expedition abroad. At the end of the year the company will be liquidated and the assets divided among the investors. This is a standard two period model. After the creation of the company there develops a market in the stock of the company without provisions for short selling. Let us examine what determines the price of these shares.

Assume the investors seek only to maximize the present value of their investment. Each investor makes the best estimate he can of the returns from the investment, and if he thinks this exceeds the return from buying government bonds he buys one share[2] in the company. Given the uncertainty about the true return to the investment in the security, potential investors make different estimates of

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1. The model used here resembles one developed by Williams and described by Smith (although it was developed independently). A more mathematical formulation is provided by Lintner (1962). They did not consider the relationship between risk and divergence of opinion, which is the key to the argument used here.

2. The limitation to one share simplifies the presentation by removing the need to discuss how large an investment is made. It does not significantly affect the conclusions of the argument. There is, of course, an intensive as well as an extensive margin in the market for securities, and fluctuations in the demand for a security can result from investors in a security changing the size of their desired holdings as well as changes in the number of investors holding a security.

expected returns from the investment. Curve ABC in figure 1 is a plot showing the cumulative distribution of the number of investors with estimates above a certain value for the amount received at liquidation of the investment. It can also be interpreted as the number of shares investors are willing to hold at each price.

Suppose that any single investor is able to purchase only one share (perhaps because of limited funds) and there are N shares available. It is clear that the shares will end up being owned by the N investors with the highest evaluation of the return. From curve ABC it can be seen that there are N investors who estimate the final value to be R or above. The selling price of the stock will be R. If it was lower there would be more than N investors who wished to hold the stock, and bidding against each other they would soon bid the stock up to R. If the price was above this level some of those holding the security would feel it over valued and would attempt to sell their share, driving the price back down to R.

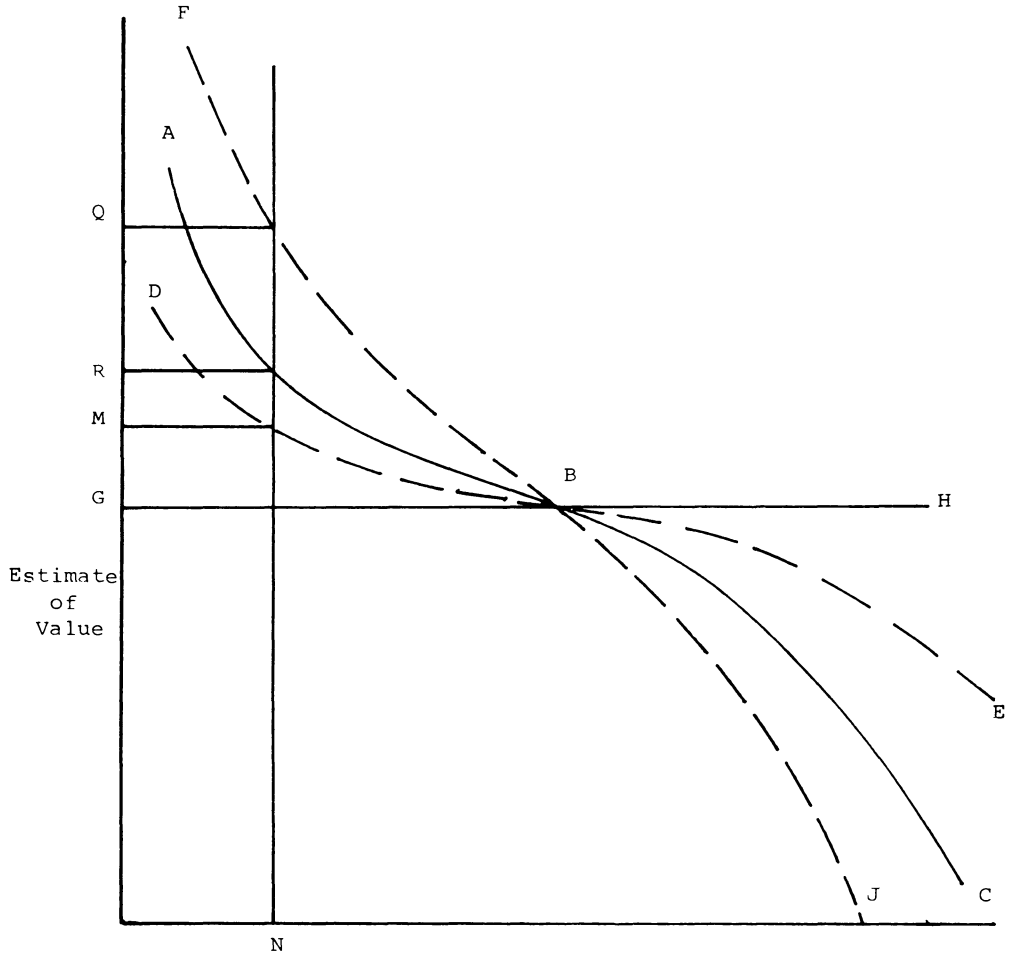

FIGURE 1.

The curve *ABC* in Figure 1 is a demand curve for the security. The supply curve is a vertical line at the number of shares available. The price is determined by the intersection of the demand and the supply curves.

Several results follow from this rather simple model. As long as the entire supply of the security can be absorbed by a minority of the potential purchasers (as is typically the case), the market price will be above the mean evaluation of the potential investors.[3]

Also, as long as a minority of potential investors can absorb the issue, an increase in the divergence of opinion will increase the market clearing price. This can be seen by noting that if curve *ABC* is replaced with curve *FBJ*, representing a greater divergence of opinions about the security, the market clearing price rises from *R* to *Q*. On the other hand, if the divergence of opinion decreases, causing curve *ABC* to be replaced with curve *DBE*, the market clearing price falls from *R* to *M*. In the limit, where there is no disagreement about the return from the security, curve *ABC* becomes the straight line *GBH*, and the market price falls to *G*. Only in this case is the market price determined by the average evaluation of the potential investors.

If potential investors make estimates of the distribution of mean returns from possible investments that are unbiased (and most economic theory assumes they do), and make investments in accordance with now standard portfolio theory, the market price will exceed the willingness to pay of an investor with perfect information about the probability distribution of returns. Hence, security markets may not produce Pareto optimal results. If firms make their investment decisions on the basis of maximizing the market value of their stock, there will be over investment in the industries and firms about which there is the greatest divergence of opinion. However, there are sophisticated investment strategies which use risk premiums to correct for the bias described above. These will be discussed later.

Before continuing the discussion of security pricing, note that security markets will produce investment decisions that differ from those produced by typical bureaucratic decisionmaking procedures (either governmental or corporate). Investment decisions made through security markets will reflect the opinions of the optimists, while those made bureaucratically will reflect the average evaluation. The market will select all investment projects for which investors are willing to take enough shares to finance the project. An investment project may be built even though the average investor is quite negative on it. In contrast, if decisions are made by having projects either accepted or permanently rejected by a single bureaucrat or a committee utilizing majority voting, the types of projects approved will reflect average opinion. The use of a committee with majority voting will reduce the spread of decisions about those that would be made by the typical decisionmaker, and this reduction will tend to increase with the size of the committee. However, if project proponents are allowed the opportunity of present-

3. As the number of investors in each stock is very small in relation to the universe of investors there is little reason to doubt that the stock is held by people who have above average estimates of returns and avoided by investors with average or lower estimates, whatever the shape of the underlying distribution of expected holding period returns by investors.

ing their proposal to a number of bureaucrats, each of whom can accept it, the results will resemble more closely those of a security market.

Speculative Excesses and the Efficient Market Hypothesis

The argument of this paper may explain how certain speculative excesses occur in which a small company is driven up to what appear to be unreasonable heights (with the benefit of hindsight). Given the low level of short selling, a badly informed or excessively optimistic small group of investors can bid a stock up to a value that most investors regard as unreasonable. A market with a large number of well informed investors may not have any grossly undervalued securities, but if these investors are unwilling to sell short (as they often are) their presence is consistent with a few investments being overvalued. (See Malkiel 1973 for examples.)

The investor who is optimistic enough about a particular stock to hold it in his portfolio will normally believe that it promises substantially better performance than most other securities available. The same analysis that led him to that conclusion is likely to persuade him that the warrant is an even better buy, (because of its superior leverage) unless the warrant is already substantially overpriced in relation to the common (considering historical violatility). This would suggest that warrants frequently will be overpriced. Since a warrant will be priced at intrinsic value upon expiration, short selling of warrants near expiration should promise supra-normal profits. Indeed, several books (Thorp and Kassouf, Noddings, Prendergast) have been written describing the extraordinary profits possible through short selling of warrants near expiration (hedged by owning the stock). Any tendency for warrants to be over-priced would suggest that firms trying to raise capital would find warrants a useful mechanism, one that would be frequently superior to common stock (assuming that issuance of warrants does not create a tax liability if the warrants expire unexercised.)

Contrary to the efficient market hypothesis, the application of skilled analysis should be able to improve performance (at a given level of risk) over that achieved by uninformed investors or random selection. Security analysis would be used not to discover undervalued securities about to undergo a rapid price increase (an activity which competition should prevent from yielding appreciable returns over cost), but to avoid purchasing (or to sell if already owned) the occasional overvalued security which less informed investors have bid up. This rather conservative, low turnover strategy to "beat the market" appears to differ widely from that currently employed on Wall Street. (See Miller, December 1976).

Risk

The above analysis has shown that the price of a security is higher the greater the divergence of opinion about the return from the security. In practice, uncertainty, divergence of opinion about a security's return, and risk go together.

Knight has drawn a distinction between risk and true uncertainty. Risk occurs where the future is not known, but the probability distribution of possible futures is known. Uncertainty occurs where the probability distribution is itself unknown.

Not much attention is paid to this distinction today because in either case the future is unknown and decisions must be made using the individual's subjective estimates of the relevant probabilities. However, the distinction does have some relevance for the subject of this paper. Occasionally, it is possible for everyone to agree on the probability distribution of possible future outcomes. A good example would be casino gambling. Here is risk without uncertainty or divergence of opinion. However, the outcomes of most investments are subject to true uncertainty, and such uncertainty normally implies divergence of opinion. In the remainder of this paper, no distinction will be drawn between the two terms and risk will be taken to imply uncertainty in accord with current usage.

In general, where there are numerous events which can make the return higher or lower than expected (risk), there will be widely differing evaluations of how likely these events are and a divergence of opinion. This leads to the surprising result, that with risk neutral investors that the expected market price for the security will increase with the risk. Indeed, even a moderate degree of risk aversion by investors might be offset. This may explain a number of puzzling results that have been reported.

Haugen and Hines have shown for a large sample of portfolios that the mean return shows a statistically significant, negative correlation with the standard deviation of their monthly returns (as well as a negative correlation with a measure of systematic risk, beta) over most five year periods, as well as over two longer periods. Soldofsky and Miller, as well as Pratt, have reported that the riskiest classes of stocks had lower ex post returns than less risky classes. This is contrary to the theory that the investor should be rewarded for assuming risk by a higher rate of return. These results can be explained by the above theory if the riskiest stocks are also those about which there is the greatest divergence of opinion. The divergence of opinion hypothesis may also help to explain the preference for price variability which has been found in several cross-sectional studies (Benishay, Chung, Bell, Bower and Bower). A plausible explanation is that the stocks with the more variable prices were the ones about which investors had the greatest divergence of opinion.

The wider the range of estimates of value made by different potential investors the more steeply sloping will the demand curve be. This is illustrated in Figure 1. In practice, the short run supply of a stock is fixed[4] so that fluctuations in price are caused by rightward or leftward shifts of the demand curve. If the frequency of shifts in the demand curve are the same for different stocks, the stocks with the most steeply sloping demand curves will have the greatest fluctuation in price.

Changes in Risk and the New Issue Market

If the divergence of opinion about a stock changes (leaving the average evaluation unchanged) the market price should change. For instance, if risky stocks become less risky over time, their prices should drop. There is some reason to believe that the stocks about which there is the greatest divergence of opinion at any one time tend to have this divergence of opinion narrow over time. This is primarily because

4. Except to the extent that short selling increases the effective supply.

the passage of time often resolves certain uncertainties about the future of a company. When this happens the market valuation will tend to decline relative to the securities whose yield is more easily estimated. This provides an additional reason for the returns on the riskiest securities to be lower than on less risky securities.

Although new issues frequently show a sharp spurt in price immediately following issuance due to underpricing by the underwriter (McDonald and Fisher, Reilly and Hatfield, Friend and Longstreet), this is followed by smaller price appreciation than "seasoned" stocks show when measured over a period of one to five years. (Stigler; Shaw; Shaw and Herman; McDonald and Fisher; Friend and Longstreet) Standard theory holds that informed investors should have foreseen the future prices of the new issues and prevented them from being bid up to the prices they achieved after issuance.

However, the theory developed here can provide a plausible explanation for the observed price behavior. The prices of new issues, as of all securities, are set not by the appraisal of the typical investor, but by the small minority who think highly enough of the investment merits of the new issue to include it in their portfolio. The divergence of opinion about a new issue are greatest when the stock is issued. Frequently the company has not started operations, or there is uncertainty about the success of new products or the profitability of a major business expansion. Over time this uncertainty is reduced as the company acquires a history of earnings or lack of them, and the market indicates how it will value these earnings. In a few cases the uncertainty is eliminated by the new firm going out of business. With the passage of time and the reduction in uncertainty, the appraisal of the top x percent of the investors is likely to decline even if the average assessment is not changed. This would explain the poor performance of a group of new issues when compared to a group of stocks about which the uncertainty does not decrease over time.

Incidentally, if underwriters ignore the above arguments and price new issues on the basis of their own best estimates of the prices of comparable seasoned securities, they will typically underprice new issues. The mean of their appraisals will resemble the mean appraisal of the typical investor, and this will be below the appraisals of the most optimistic investors who actually constitute the market for the security. This may be a partial explanation for the underpricing of new issues by underwriters.

Systematic Risk

A key question is whether systematic risk and uncertainty normally occur together. It is known that stocks that are risky using the criteria of high covariance of their return with the market return also tend to be risky using other criteria (Miller and Scholes; Malkiel and Cragg; Klemkosky and Martin; McEnally). Since much stock trading consists of investors who are pessimistic about a stock selling to those who are optimistic, turnover provides one measure of diversity of opinion (strictly speaking of changes in relative opinion). Cooley and Roenfeldt have shown that high turnover is typically accompanied by lower returns, a result consistent with the hypothesis of this paper. Ben-Zion and Shalet, and Rosenberg and McKibben have shown a positive correlation between turnover and beta suggesting that there

is indeed more diversity of opinion about high beta stocks than about low beta ones.

A number of investigators (Beaver, Kettler, and Scholes; Hamda; Breen and Lerner; Rosenberg and McKibben) have shown that stocks that are considered to be risky by traditional financial criteria tend to be high beta stocks. This is partially because optimism and willingness to bear risk increase during periods of high stock prices, and partially because traditional measures of risk indicate how vulnerable the company is to the business cycle (which is correlated with the price level of stock).

If systematic risk and uncertainty normally occur together and if investors act on their own best estimates of expected risk and return, the price of risky securities will be raised (and their yield reduced). A logical result will be that the returns on risky stocks (high beta stocks) will be lowered below what would be predicted by the standard capital asset pricing model. In particular, the riskiest stocks will have a yield that is below the capital market line connecting the risk free interest rate and the market portfolio.

Interestingly, such a result has been reported by several authors including Petit and Westerfield (1974), Friend and Blume, Sharpe and Cooper, Fama and McBeth, and Black, Jensen, and Scholes. These studies have shown, using portfolios selected from stocks on the New York Stock Exchange, that the return on high beta portfolios was less than that achievable by borrowing money in order to hold a leveraged portfolio of lower systematic risk stocks. In particular, the intercept on the return axis of a plot of return versus beta for different portfolios exceeded the risk free interest rate. The above theory provides a plausible explanation for these otherwise puzzling results.

Specification Bias

Expectations are of critical importance to most theories of security markets. A standard research methodology is to assume that realized returns are a good surrogate for the expectations of investors. The argument of this paper shows that prices do not reflect expectations of typical investors, but of the minority who buy the particular stock. As a result ex post investment results cannot be used to measure ex ante investor expectations for the average investor. This appears to be what happened to the tests of the capital asset pricing model discussed above. Brown (1974) has described a similar bias applying to ex post evaluations of investment projects. Dhingra has pointed out that parameter estimation errors will cause investors to experience more risk and less return than expected.

A Risk Neutral Rationale for Avoiding Risky Investments

So far this paper has assumed a straightforward method of security evaluation and portfolio selection. The investor uses standard security analysis to estimate the returns and risk of different investments, then uses portfolio theory to select an efficient portfolio. This is the model usually assumed in the literature. It was argued that it would typically result in investors realizing lower returns on the securities for which the returns were more uncertain and riskier. This in turn

suggests that investors would either not use the obvious approach given above, or would soon abandon it.

For instance, an investor might notice that he was earning less on his riskier investments than on his less risky investments even though he had tried to apply the same criteria to both. Even if the investor was neutral towards risk he might start evaluating risky projects using a discount rate that is higher than he uses for less risky projects. Hence, a risk neutral (or even risk preferring) investor may use a risk premium (really an uncertainty premium) in evaluating projects to adjust for the higher market prices of risky projects. Thus, the use of risk premiums in evaluating projects does not necessarily show that the investor really is risk averse.

The problem of the manager of a diversified company considering investing in a new production complex directly is logically equivalent to that of a portfolio manager considering buying a large block of stock in a new company set up to build such a production complex. Just as the portfolio manager should employ uncertainty premiums in evaluating portfolio investments, businessmen contemplating investment where there is uncertainty should use uncertainty premiums in evaluating physical investments. (See Miller, February 1977.)

Similar effects occur in competitive bidding. In bidding for oil leases, the firm that is most optimistic about the prospects for finding oil will tend to be the high bidder for the lease[5] (Miller, 1969, 1972). In bidding on construction contracts, the firm most optimistic about the costs of construction will tend to win. If estimates are unbiased on average, the winning firm will typically be one whose errors were such as to overestimate potential profits. Consistent losses can only be avoided by employing an uncertainty premium. This is one reason construction contractors have found that it pays to employ a liberal "contingency allowance."

Finally, if enough investors use uncertainty premiums in evaluating investments, the demand for risky investments will be reduced, and their prices will tend to fall. The final effect could be to cause investments about which there is a wide divergence of opinion (high uncertainty about return) to sell at a price which is close to the return expected by the typical participant in the market (or below if enough participants in the market are averse to risk). While such an outcome is quite possible, it is not the result of market participants naively estimating the expected return and trading off the level of expected return against risk as assumed in the standard capital asset pricing model. The absence of a literature (except in the context of competitive bidding[6]) dealing with optimal strategy where there is divergence of opinion suggest that many investors are still following naive procedures.

Use of Risk Premiums for Evaluating Public Projects

Just because risk premiums are commonly used by private investors does not mean that public investments should be evaluated using similar risk premiums as would

5. The idea for this article resulted from generalizing this observation to the stock market where the investor who is most optimistic about a stock typically wins the bidding and his evaluation sets the price.

6. See Rothkopf and his references, and Copen, Clapp, and Campbell.

be used by a private investor looking at securities of companies undertaking the same or a similar project. Private investors in securities may use risk premiums because they are engaged in a game against other investors. The greater the uncertainty the wider the divergence of opinion, and the lower the ex post return from bidding for a security on the basis of one's own estimate of expected return.

Public projects are games against nature, and the cost of building a project does not normally increase with the divergence of opinion about the project. The optimal evaluation strategy may differ for investors in public projects (or projects to be built by the investor) and for investments where the investor is considering buying in a competitive market, bidding against other investors.

However, a major difference between the typical public investment and the typical private investment is that owners of private securities can sell them in time of need, while the typical claim to the benefit of a public investment is non-marketable. Thus private securities reduce risk by providing liquidity in a way that most public investment cannot. As a result a rational investor will normally require a much lower return from a private investment than he would require from a illiquid public investment. The difference can be viewed as a different sort of risk premium (a liquidity premium). (See Miller, October 1976.)

Relaxing the Assumptions

In the initial discussion the investor was assumed to be risk neutral for simplicity of exposition. This can be relaxed without changing the argument since the individual investor is assumed to use the standard portfolio selection theory (Markowitz, Sharpe, Lintner) which allow for risk aversion. Because uncertainty produces both risk and divergence of opinion, the incorporation of divergence of opinion into a model with restricted short selling reduces the risk premiums on the riskier securities, and may even make them negative.

Using his own best estimates of the mean return and distribution of returns from various investments, each investor finds his efficient frontier, and then selects the point on the frontier that brings him to the highest point on his indifference curve. With non-homogeneous expectations the set of risky securities included in the efficient portfolio will differ among investors, as will the amount of securities held. For equilibrium in the security market the prices of all securities must be such that the desired holdings sum to the total amount outstanding of each security. If they do not, attempts at buying and selling of securities will cause prices to change in the direction of equilibrium.

If investors desire to hold at the initial set of prices less than the amount outstanding of a particular security, selling pressure will cause the price to drop. As it drops the security will enter into the portfolios of more and more investors; normally those who have the most optimistic beliefs about its expected return and risk. Holding the prices of all other securities constant, curves such as those of Figure 1 can be derived. (Figure 1 now reflects not only the larger number of investors holding a security at lower prices but also the increase in the number of shares held by each investor.) The price of the security will fall, until investors are just willing to hold the quantity in existence. On Figure 1 this is where the vertical line representing the total quantity issued cuts the curve. Equilibrium will normally

be reached with the security included in the portfolios of only a small fraction of investors. This implies that the price of the security will be above the estimate of the value made by the average investor. (Here, the investor's valuation is taken to be the highest price at which he would include any of the security in his portfolio.) The price at which the entire issue will be absorbed will increase with the divergence of opinion about the return to be expected from the security. The typical stock will be on the efficiency frontier of the optimists at prices which would cause it to be inside the efficiency frontier of the typical investor (who would avoid it as being dominated by other investments). Thus, even in a situation where there are a number of risky investments which may be held in variable quantities the basic argument holds.

Short Selling

Short sales permit an investor to create stock in a company by agreeing to pay to an owner of the existing stock any dividends declared by the company, and to redeem in valid stock the borrowed shares upon demand. From the viewpoint of the holder of the stock that is borrowed, this created stock is equal in all ways (except for voting power) to the originally issued stock and satisfies fully his desire to hold stock in the company. (Indeed the lender will not normally know that his stock certificate has been loaned out by his broker.) The effect of the short seller on the supply of a particular stock is exactly analogous to the effect of a bank on the supply of money. The bank borrows currency agreeing to replace it upon demand and lends it to a third party. The depositors act as if they still had possession of the money loaned since they can regain it upon demand. They neither know nor care if their deposited currency is loaned out.

The result is that short sales increase the supply of stock on the market by the amount of the outstanding short position. On Figure 1 the effect of short sales is to move the vertical supply curve to the right by the amount of the outstanding short position, lowering the price.

This causes the existence of adverse opinions to affect the market value of that stock. Because the number of people with extremely pessimistic evaluations of a stock are likely to increase with the divergence of opinion about a stock, short sales tend to moderate the tendency for riskier stocks to be bid up to higher prices.

Short sales can only moderate the bidding up of riskier securities because short selling is profitable only with stocks that decline in price at a rate sufficient to cover the dividends the short seller owes to the lender of the stock. It is impossible to make a profit by selling short an overpriced stock which will have a subnormal return, if that return is positive. The reason is that the short seller does not receive use of the proceeds of the short sale (or even interest on them). Instead they are deposited as an interest free loan with the lender of the stock, providing him with security for return of the stock and compensation for making the loan. This market imperfection means that even large numbers of investors willing and able to sell short will be unable to insure an efficient market in the presence of heterogeneous expectations. (See Miller, December 1976.) Among other implications this, along with the presence of at least one badly informed investor, detroys the theoretical case for the random walk hypothesis.

The failure to allow the short seller use of the proceeds of his short sale also has implications for the pricing of options and warrants. The well-known Black and Scholes formula (1973) gives the necessary prices for an option if there are not to be opportunities for risk free hedges yielding more than the riskless rate of interest. It was derived under the unrealistic assumption that the short seller receives the proceeds of the short sale at the time of the sale. This assumption is true for covered sales of exchange listed options, but is not true for short sales of stocks or warrants. If short sellers do not receive immediate use of the proceeds of their short sales, there is a range of values over which the prices of options and warrants can fluctuate without providing possibilities for above normal profits through hedging. (For descriptions of such strategies see Thorp and Kassouf, Noddings, Prendergast.) The lower price limit for warrants will be the price at which it is possible to earn a normal rate of return through a hedge constructed by buying the warrants and selling short the common stock. The upper limit will be the price at which it is possible to earn a normal return through buying the common stock and selling short the warrants (so as to set up a risk free hedge). This upper limit may not exist if there is a risk of the warrants being extended for tax reasons, if the low price of the warrants leads to them being unavailable for borrowing in sufficient quantities, or if the exchange forbids further short sales because of the large short interest already outstanding.

It was argued that in the absence of short selling heterogeneous expectations would lead to warrants being overpriced. The same argument suggests that warrants are likely to trade at the upper limit of the above range more often than at the lower limit. Thus, even if short selling is permitted, warrants are likely to be over-priced. Their expected returns will typically be positive, but subnormal.

Options are not normally sold short. However, the investor can achieve the same result by writing covered options. Here the writer (short seller) receives immediate use of the proceeds of the sale. Thus, the possibility of constructing a hedge by buying stock and writing options should place an upper limit on the prices of options at a level that would be near the Black-Scholes price (disregarding transaction costs). The lower limit would result from hedges involving short sales of the stock combined with purchases of the option. If option prices were uniformly distributed between the two limits, one could conclude that options would normally be slightly underpriced. Unfortunately, the frequent commissions involved in a strategy of remaining invested in options would probably destroy any advantages over buying and holding stocks. Moreover, the same argument applied earlier to warrants leads to the conclusion that options will normally be at the upper limit of their range, eliminating the underpricing.

Short selling does not play a role in the standard capital asset pricing model because the assumption of homogeneous expectations implies that an investor holds an identical proportion of the total issue of all risky securities. This implies that risk averse investors will desire to hold only positive quantities of securities. Since risk averse investors will not go short, even if given the option, a homogeneous expectations model with restricted short selling gives exactly the same conclusions as one with unlimited short selling as long as investors are risk averse or risk neutral (risk preferring investors may desire to go short if permitted).

However, this is not true for non-homogeneous expectations. As described above, without short selling the price of a security is raised if there is divergence of opinion. A sufficient amount of short selling could increase the volume of the security outstanding until its price was forced down to the average valuation of all investors. This effect is essential to Lintner's (1969) model with heterogeneous expectations and unrestricted short selling, which gave results similar to the traditional capital asset pricing model. In theory, there is no limit to the number of shares of stock that can be created through short sales. (There is nothing equivalent to the required reserve ratios that limit the growth of the money supply.) However, in the real world there are restrictions on short selling, and the number of short positions is usually only a small fraction of the total number of shares outstanding.

Since the stock created by short sales competes with stock issued directly by companies, restrictions on short sales could serve to make equity financing easier. This might be at the cost of somewhat greater fluctuations in stock prices, and less realistic prices for speculative securities.

Implications for the Market Valuation of Multi-Industry Companies

The existence of non-homogeneous valuations of securities among investors has implications not only for the prices of particular securities, but also for the valuation of firms formed by mergers, of conglomerates, and of closed end investment companies.

Imagine that two firms in different industries merge. For convenience in exposition make one a steel firm and one a meatpacking firm. To avoid bringing in extraneous factors assume that the financial structure, riskiness, price-earning ratio, etc. of the firms are the same. What will be the effect of the merger on the total market valuation of the merging companies? The combined firm would be more diversified and would have a more stable income. This has led some to argue that it should have a higher market value than the sum of the values of the merging firms. However, several recent writers (Schall, Levy and Sarnat) have noted that a conglomerate merger is likely to change neither the total income nor the systematic risk of the merging companies, and that in the absence of changes to one of these variables, the total market value should not be affected. Anyone who desired a portfolio that included both the steel and the meatpacking firm could have it prior to the merger merely by buying shares in the two companies separately. The result would be that a merger should leave the market value of the companies unchanged.

The model sketched in this paper leads to the conclusion that the total market valuation will be lowered by a merger. The price for steel company shares is set in a market in which the major buyers are those who are optimistic about the steel industry. Most of these purchasers are unlikely to be as optimistic about the merits of meatpacking investments. When they make their best estimates of the future earnings and dividends (per dollar invested) of the combined meatpacking-steelmaking firm the estimates will tend to be lower than the corresponding estimates for firms only in the steelmaking business. They are likely to conclude that a firm only in the steelmaking industry represents a better investment. A similar analysis can be applied to those investors who have high opinions of the

investment merits of meatpacking. They are likely to conclude that an investment in a meatpacking-steel company is less desirable than one in a pure meatpacking firm. The result is that neither those optimistic about steelmaking nor about meatpacking will be interested in the merged firm at the pre-merger prices. This will cause a fall in the price of the merged firms' stock until the stock becomes attractive to those who have more normal expectations for one of the industries.

A possible explanation for the low prices of conglomerates[7] and closed end investment companies (for examples see Fishbein; Malkiel 1973 Appendix 3) is that the typical investor finds that such investments are dominated by investments in a single industry, or related group of industries using whatever criteria he himself used for ranking. The preferred investments, of course, are not the same for all investors: depending on their evaluations of the potential returns a wide range of other companies may prove to be the preferred investment. The result is that it is possible for the typical security analyst to argue that conglomerates or closed end investment companies are under-valued, but yet not choose them for his portfolio because there are companies in particular industries that are even more under-valued. (For alternative explanations see Malkiel 1977.)

Some support for this can be seen in the literature oriented towards potential investors. It is common to see a discussion of the excellent prospects for a particular business, followed by a discussion of which companies participate in the business, and the fact that the typical firm derives only a small fraction of its sales from that business. Certain companies are then recommended as being the "purest play" available, and their stock may be driven up. For instance, the discovery of the Prudhoe Bay field in Alaska drove the price of the relatively small Atlantic-Richfield up by more than that of the gigantic Exxon, even though both had identical half interests in the discovery (Norman).

There exist a number of methods of security evaluation, frequently based on different attributes of a security: dividend yield, book value, price-earnings ratio, growth prospects, etc. Different investors tend to use different methods. Ownership of a security will gravitate towards the investors who employ the evaluation methodology which is most favorable to it, and the price will tend to be the value assigned to it by investors relying on that methodology. For instance, high dividend paying stocks tend to be owned by those who evaluate stocks on the basis of dividend rates, and their values tend to be based on valuation methodologies which heavily weight dividend rates. "Growth stocks" tend to be owned by those who evaluate stocks on the basis of their growth prospects, and their values will be set by their growth prospects. One implication for those who try to devise econometric models is that models which are linear functions of various attributes of stocks are likely to give relatively poor fits. Better results may come from models which choose the maximum value given by any one of a set of different formulas. Such models might be developed by dividing stocks into groups which appear to be priced on a similar basis, and then estimating the pricing function applicable to that group of stocks.

7. Conglomerates once sold at a premium but this disappeared when their accounting became better known and their rate of acquisitions slowed.

Financial managers of companies should determine which evaluation method is being used by those investors who constitute the market for that companies' stock, and then attempt to give them what they value. Sometimes this may call for splitting a company up. For instance, a company may have some money losing divisions, some divisions showing normal profits in a stable business, and one or more divisions with little current profits, but great prospects (or at least hopes). Most likely the market will value the company by applying a moderate price earnings ratio to the current earnings of the company. The total value of the stockholders' investment might be maximized by splitting the company into several parts, and letting the value of each part be set by those investors who value its attributes. In such a nongrowth company, the divisions with great prospects (or what certain groups of investors would see as great prospects) contribute little to the value of the company except their current earnings. If separated they could sell as growth companies. The money losing divisions probably reduce the value of the parent company by lowering its current profits. If separate they could be seen as possible turnaround situations or asset plays. (They could not possibly have the negative value they have as part of the parent company.) The remainder of the parent company would continue to sell at a normal multiple of its now increased earnings. This explains why companies frequently sell money losing operations even though the new owners have no better chances of turning the operations around than the original owners.

The chief deterrent to spinning off parts of companies (besides the self-interest of management) is that a significant part of the investing community appears to value size for its own sake. After a spin-off, none of the parts are as big as the whole was. Certain parts may be too small to appeal to institutional investors or to very small, conservative individual investors (for whom transaction costs encourage concentration in a few large, safe diversified companies).

In addition to differences in the evaluation of particular securities, investors differ in their investment needs. For instance, those whose income is sensitive to the business cycle will seek to avoid those securities whose capital value is closely correlated with it. The parts of a conglomerate or holding company may appeal to different classes of investors, and be worth less when tied together than when offered separately. For instance, the value of a "cyclical" security depends on whether the investor is holding it as a reserve to finance consumption during a depression induced period of low income, or for some other purpose (Miller 1977).

The Visibility of a Security

So far the model has been developed with the assumption that each investor investigates all securities, and then picks the ones that contribute the most to his portfolio. Realistically, there are more securities available than the typical investor can evaluate. The probability that a particular stock will be purchased by an investor is the probability that he will investigate a stock and the conditional probability that after investigation he will decide to include it in his portfolio.

This raises the possibility that stocks may differ in the extent to which they are investigated, and that this will have effects in the market. Since all existing stock must be held by someone, any decrease in the fraction of investors interested in the

stock of a company must be offset by an equivalent increase in the fraction of those interested who decide to include it in their portfolio. The price of the stock must fall to increase the fraction of the investors who, after evaluation, include it in their portfolio.

For instance, if absorption of a security issue requires that 1% of the investors hold it, the price will be set by the most optimistic 1% of the investors, if all investors look at it. However, if only one out of ten investors ever look at it, the price will be set by the most optimistic 10%. Obviously, this effect will be most important for those stocks about which there is the greatest divergence of opinion, which will tend to be the riskier stocks. It should be realized that the fraction of investors looking at a stock is probably not independent of its price. If the price falls too low the financial press and the brokerage industry may publicize its apparent undervaluation, causing more investors to look at it.

There are several reasons why the amount of investor attention to a stock may differ. Some companies are naturally well known because their products are widely advertised and widely consumed (Coca-Cola). Companies of the same size that sell raw materials and capital goods to other companies or that sell under someone else's brand names (suppliers of private branded goods) will be less well known. This would suggest somewhat higher prices for consumer goods companies. The author's casual observation of the market suggests that such higher prices exist. Testing of the above hypothesis is complicated by the tendency for the profits of consumer goods companies to fluctuate less with the business cycle than those of raw material or capital goods companies. The result is that such companies tend to have less risk and lower betas than other companies.

Within the category of consumer goods companies, those selling to higher income consumers (makers of luxury goods or "quality" brands of other goods) should be better known to those individuals with funds to invest than those selling to the working classes. The author's casual empiricism would suggest some truth to this hypothesis.

Of course, the awareness of a security may be increased if the issuing company receives much publicity. For instance, new products and technological breakthroughs are news so that companies producing such products receive more publicity. Also, companies which undergo frequent changes in fortune frequently receive a disproportionate amount of attention in the media. Needless to say such companies tend to be among the more risky ones. This is another possible contribution to the poor performance experienced by the riskiest companies. It should be noted that publicity need not be particularly favorable for it to attract attention to a stock. Some of those led to investigate a stock may choose to buy it. Since existing holders of a stock are likely to be following the stock, merely neutral publicity is unlikely to cause much selling.

Consider a stock held by 1% of investors. A particular article is likely to cause 10% of the readers to lower their opinion of the stock sufficiently to cause them to sell one share of the stock if they held any, and to cause 1% of readers to purchase a single share. Although such an article would probably lower the average assessment of the market, the net effect is induced purchases 10 times as large as the induced sales.

This is a result of asymmetric behavior of those who do not hold the stock and those who do combined with holders of the stock typically being in a small minority. Published information on a stock may cause existing holders to either increase or decrease their holdings, while reluctance to make short sales limits the response of most non-owners to purchasing the stock, or doing nothing. Due to divergence of opinion and transactions costs, the holders of a given stock are typically only a small minority of the total investor population. They are vastly outnumbered by the non-holders, who can only come in on the buying side. Thus, it is hypothesized that anything that increases investor awareness of a stock will increase its price.

The above may explain why in pre-Securities Exchange Commission days pools would attempt to increase volume by buying and selling a stock at the same time. In theory, high volume does not indicate that the stock will rise (it may be caused by heavy selling), and merely observing heavy volume should not cause anyone to buy. However, if the volume does attract attention and cause more people to look at a stock, some are likely to persuade themselves that the stock should be bought. The final effect is then to attract some buying of the stock, allowing the pool to unload at a profit.

Conclusions

In practice, uncertainty and risk imply divergence of opinion. In a market with little or no short selling the demand for a particular security will come from the minority who hold the most optimistic expectations about it. Since divergence of opinion is likely to increase with risk, it is quite possible that expected returns will be lower for risky securities, rather than higher. Even for risk neutral investors, optimal strategies will involve the use of risk premiums in evaluating securities. The presence of a substantial number of well informed investors will prevent there from being substantially undervalued securities, but there may be securities whose price have been bid up to excessive levels by a badly informed minority, thus contradicting the efficient market hypothesis.

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