The model in Table 8.1 shows the basis of how a concern for a relative measure of wealth generates the equivalent riskiness to low and high beta assets. Assume there are two assets, X and Y, and two states of nature, 1 and 2. An investor faced with asset X or Y can see the following payoffs.

Y is considered riskier in standard theory, as shown in Table 8.1, with a 40-point range, versus a 20-point range for X. Yet on a relative basis, each asset generates identical risk. In State 1, X is a +5 outperformer, in State 2, X is a −5 underperformer, and vice versa for asset Y. In relative return space, the higher absolute volatility asset is not riskier, and the careful reader can check this for any example where the two assets have the same mean payout. If X and Y are the only two assets in the economy, relative risk can be achieved, equivalently, by taking on an undiversified bet on X or Y. Buying the market, or allocating half of each, meanwhile, generates zero risk relative to the mean portfolio.

The absence of a risk-return relation flows from this simple insight, and while one can prove this various ways using sophisticated mathematics, it really is a simple idea from assumption to implication. The idea that people benchmark is the key driver, because ubiquitous benchmarking is the same thing as having a relative utility function. When relative portfolio wealth is the argument in the utility function, any relevant volatility is symmetric, because the complement to any portfolio subset will necessarily have identical—though opposite signed—relative volatility. You can prove, using utility functions or arbitrage, that if people care about their relative return, the risk premium is zero. The arguments are no simpler, and use no fewer assumptions, than what is necessary and sufficient to generate the traditional risk premiums. Simply replace the utility function argument of wealth, with relative wealth, and no risk premium. The driver of the result is driven...
TABLE 8.1 Relative Risk Is Symmetrical

<table>
<thead>
<tr>
<th></th>
<th>Total Return</th>
<th></th>
<th>Relative Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
<td>Avg.</td>
</tr>
<tr>
<td>State 1</td>
<td>0</td>
<td>-10</td>
<td>-5</td>
</tr>
<tr>
<td>State 2</td>
<td>20</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>

by the logic described here, because like idiosyncratic risk in the traditional theory, anything that can be arbitaged, either will be arbitaged (in arbitrage theory, by assumption), or is implicitly arbitaged (in utility, an equilibrium exists only when there are no gains from trade remaining).

Furthermore, note the symmetry of risk. The *sine qua non* of particle physics is symmetry, and I’m not saying this necessarily implies truth, but symmetry is an attractive quality by itself. Risk is symmetrical because risk is betting against the consensus, and that implies someone thinking the exact opposite. Thus, my risk to buy X cannot have a risk premium because my complement’s position—to sell X, has just as much risk, but is on the other side. Ergo, there can’t be a risk premium. If they were both risky, then you could create a portfolio of equally risky positions, long and short for the same security in the same amount, generating a positive return. This is an absurdity because it means you get a positive return for having a net zero position, in regard to both cost and volatility.

An important point is that for idiosyncratic risk, the results will be identical to the orthodox approach. That is, a chance to win the lottery, or bet on a horse race, or experience a car crash, is independent of what most everyone else is doing, and thus is just as risky, and unpriced (other than its expected value). Thus, a general aversion to uncertainty, or volatility, is implied in this approach just as the standard approach, because idiosyncratic volatility lowers utility when compared to a benchmark, or when merely applied to one’s nest egg alone.

**BENCHMARK RISK**

Even the main champions of the standard model in effect acknowledge that as a practical matter, risk is relative to a benchmark. Bill Sharpe consults for pension funds evaluating asset managers and states that his first objective is “I want a product to be defined relative to a benchmark.” Fund Manager
Kenneth Fisher’s book *Only Three Questions That Count*, in the index next to *Risk*, has “see Benchmarking.” When asked about the nature of risk in small stocks, Eugene Fama noted that in the 1980s, “small stocks were in a depression,” and Merton Miller noted the underperformance of the Dimensional Fund Advisors small cap portfolio against the S&P500 for six or seven years in a row was evidence of its risk. But the smallest 30 percent of stocks actually had a higher return in the 1980s compared to the 1970s; it was only relative to the indexes that they had negative returns. Eugene Fama notes that “Most investors are probably sensitive to the risk of being different from the market, even if overall variability is no higher. Value stocks do not outperform market portfolios regularly or predictably—if they did, they would not be riskier.” This is exactly my point. It seems reasonable to presume that for these investment professionals and academics, risk, intuitively, is a return relative to a benchmark, often a passive index. If all investors, and academics, act as if they are benchmarking to aggregate indexes, risk will not be priced in equilibrium.

For modern investors, portable alpha is the idea that alpha should be separated from beta risk, so that a fund manager with alpha may provide both alpha and beta risk (the beta risk being his risk of being long the stock market). You pay a premium for the alpha, but as beta is available at very low cost through passive funds if not futures, and you should not pay for that risk. Furthermore, to the extent your portfolio optimization algorithm has a slot for stock market beta, it would be nice if portfolio managers would all set their betas on conventional asset classes to zero, just to make the problem, and the costs of management, easier to evaluate. The net result, however, is alpha managers all being beta neutral, and so, defining risk as any deviation from a beta of 1 in their asset class. Thus, a convertible bond manager with a beta of 0.5, and one with a beta of 1.5, both have the same beta risk. In general, one does not appreciate these deviations; they only add complexity to the overall portfolio problem. There is little evidence that market timing is a large part of anyone’s alpha, and so these bets are viewed much more skeptically.

**WHY RELATIVE RISK LEADS TO NO RISK PREMIUM**

We can also see how this result holds through arbitrage if investors are concerned about relative performance. Assume an economy with risky assets that are a function of a market factor, $r_m$. For any investor $i$ who
chooses an asset with a specific beta $\beta_i$, returns are generated by the factor model

$$r_i = \mu_{\beta_i} + \beta_i r_m$$  \hspace{1cm} (8.1)

where $\mu_{\beta_i}$ is a constant for an asset with the specific beta $\beta_i$, and $r_m \sim N(\mu_m, \sigma_m^2)$. We will assume no idiosyncratic risk from assets, because the gist of this approach is better uncluttered by the extra notation.

The return on the risk-free asset is the constant $r_f$. The market price of all assets, risky or risk-free, is assumed equal to 1, so we are solving for a $r_f, \mu_{\beta_i}$ and $\mu_m$ such that this is an equilibrium.

The market return in this model is the benchmark to which investors compare themselves, just as mutual fund managers typically try to outperform their benchmark. Their objective is to maximize their outperformance, subject to minimizing its variance. Define $r^i_{out}$, which is the relative performance of investor $i$ to the market return

$$r^i_{out} = r_i - r_m$$  \hspace{1cm} (8.2)

Here $r_i$ is the return on the investor’s portfolio with its particular factor loading $\beta_i$, and $r_m$ is the return on the market. Investors all have the simple objective of maximizing $r^i_{out}$ while minimizing a proportion of its variance, as in

$$\text{Max } r^i_{out} = \frac{a}{2\sigma_i^2}$$  \hspace{1cm} (8.3)

Where $\sigma_i^2 = \text{Var}(r_i - r_m)$. Substituting equation (8.1) into (8.2) generates

$$r^i_{out} = \mu_{\beta_i} + (\beta_i - 1) r_m$$  \hspace{1cm} (8.4)

Since $r_m$ is the only random variable, the variance of outperformance is just

$$\sigma_i^2 = (\beta_i - 1)^2 \sigma_m^2$$  \hspace{1cm} (8.5)

We can replicate the relevant risk of a stock with a beta of $\beta_i, \sigma_i^2$, through a portfolio consisting of $\beta_i$ units of the market portfolio, and borrowing $(1 - \beta_i)$ units of the risk-free asset (cost is $\beta_i - (1 - \beta_i) = 1$, the same as for the stock, because all assets have a price of 1 by assumption). Arbitrage then implies that these have the same expected returns, so

$$E(\beta_i r_m + (1 - \beta_i) r_f) = E(\mu_{\beta_i} + \beta_i r_m)$$  \hspace{1cm} (8.6)
The LHS of equation (8.6) is the market portfolio levered \( \beta_i \) times by borrowing \((1 - \beta_i)\) in the risk-free asset in financing, while the RHS is the unlevered \( \beta_i \) asset portfolio by equation (8.1). They have the same factor exposure, and cost the same, so they should have the same return in equilibrium. Thus equation (8.6) implies

\[
\mu_{\beta_i} = (1 - \beta_i) r_f \quad (8.7)
\]

This allows us to replace the \( \mu_{\beta_i} \) with \((1 - \beta_i) r_f\) in equation (8.4) and leads to the factor model

\[
r_{out}^i = (1 - \beta_i) r_f + (\beta_i - 1) r_m \quad (8.8)
\]

If the degree of risk of relevance to investors is their outperformance, \( \sigma_i^2 \), the expected return for assets with \( \beta = x \) should be the same as those with \( \beta = 2 - x \), because they have the same risk in this environment:

\[
(x - 1)^2 \sigma_i^2 = ((2 - x) - 1)^2 \sigma_m^2.
\]

The risk of a \( \beta = 2 \) asset is identical in magnitude to a \( \beta = 0 \) asset, so the expected returns must be the same

\[
E(r_{out}^i | \beta = x) = E(r_{out}^i | \beta = 2 - x) \quad (8.9)
\]

Using equation (8.8) and applying the expectations operator to equation (8.9), we have

\[
(1 - x) r_f + (x - 1) E(r_m) = (1 - (2 - x)) r_f + ((2 - x) - 1) E(r_m) \quad (8.10)
\]

The LHS of equation (8.10) is the expected return on the \( \beta = x \) asset, and the RHS is the expected return on the \( \beta = 2 - x \) asset. Solving for \( E(r_m) \) we get

\[
E(r_m) = r_f \quad (8.11)
\]

Equations (8.1), (8.7), and (8.11) imply

\[
r_i = r_f + \beta_i (r_m - r_f)
\]

\[
r_m \sim N(r_f, \sigma_m) \quad (8.12)
\]

Thus no arbitrage, in the sense things equivalent in risk are priced the same (as risk is defined here), generates the traditional CAPM with the significant difference that the expected market return is equivalent to the
risk-free rate. Just as the equilibrium model in the prior section implies, the expected return on all assets is the same, because $E\beta(r_m - r_f) = 0 \forall \beta$.

In contrast, a traditional arbitrage model would take from the arbitrage equation (8.7), and, combined with the market model equation (8.1), generate the standard factor model

$$r_i = r_f + \beta_i(r_m - r_f) \quad (8.13)$$

But now the maximization function reflects the fact that the investor only cares about absolute volatility, not volatility relative to some benchmark.

$$\max_{\beta_i} r_i - \frac{a^2\beta_i^2\sigma_m^2}{2} \quad (8.14)$$

Substituting equation (8.13) for $r_i$, the first order condition on equation (8.14) generates the familiar equation

$$\beta_i = \frac{E(r_i - r_f)}{a\sigma_m^2} \quad (8.15)$$

So investor $i$'s optimal $\beta_i$ will be equal to the risk premium over the risk aversion coefficient times the market variance. Assuming a representative investor, conventional parameters for this approach of 6 percent for the risk premium, 1.5 for a risk coefficient, and 20 percent for market volatility, this implies an equilibrium beta choice of 1, consistent with an equilibrium where the representative investor holds the market basket. But if, as argued here, the market premium is in effect zero, the $\beta_i$ choice will be zero, which is not an equilibrium, because on average the market beta is 1 by definition and in positive net supply. In the traditional approach, a positive market premium is necessary for investors to hold the market in equilibrium, whereas in a relative risk model, combining equations (8.3) and (8.8), we get

$$\max_{\beta_i} (1 - \beta_i) r_f + (\beta_i - 1) r_m - \frac{a^2(\beta_i - 1)^2\sigma_m^2}{2} \quad (8.16)$$

$$\beta_i = \frac{E(r_m - r_f)}{a\sigma_m^2} + 1 \quad (8.17)$$

Here, the optimal choice of $\beta_i$ is 1 only if the risk premium is zero (that is, $E[r_m] = r_f$), because risk is uncompensated by arbitrage; risk can be avoided in this model by choosing a beta of 1. A positive risk premium would induce a desired optimal beta greater than 1, which would then not be in equilibrium.
While this is a simple model, it has at its essence no more simplicity than what generates traditional risk premiums. The only difference is whether one puts relative as opposed to absolute wealth in the utility function. Both the absolute and relative risk approach generate the familiar factor pricing model, but in the relative risk approach, the risk premium is zero in equilibrium, whereas in the absolute risk approach, the risk premium must be positive.