Minimizing Basis Risk from Non-Parallel Shifts in the Yield Curve

Part II: Principal Components

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The Journal of Fixed Income

Volume 7, Number 1

June 1997

December 1996

Introduction

In Falkenstein and Hanweck, (1996), we presented a technique for hedging fixed-income portfolios against non-parallel yield curve shifts called *covariance-consistent key rate hedging*, or more simply, *covariance key rate hedging*. The concept behind covariance key-rate hedging is to find the combination of hedging securities that minimizes the variance of changes in the hedged portfolio. The building blocks of this approach consist of the sensitivities of the portfolio and the hedging securities to changes in the key rates (partial durations or basis point values), and the covariance matrix of these rates.

We found that the covariance-consistent hedge typically provides a better hedge than traditional duration hedges and yield-beta methods. The technique is fairly simple, produces intuitive results, and is straightforward to perform in a spreadsheet. Moreover, key rate durations have become a well-accepted tool for analyzing interest-rate risk, and many risk-management systems provide key-rate durations of fixed-income instruments and portfolios. Key rate covariance matrices are available publicly or are easily estimated from historical data.

Over the past few years, another technique for hedging interest-rate risk, the method of *principal components*, has grown in popularity. This approach seeks to find a small number of components or factors that explain most of the variation in key rates, and use these factors as the drivers of the yield curve. The principal components hedge is the combination of hedging securities whose sensitivity to changes in these factors best offsets the sensitivity of the portfolio to changes in the factors.

Principal components has several appealing features. It provides a simple way to model changes in the yield curve, since only two or three components usually suffice to explain 95% or more of the variation in the yield curve (Litterman and Scheinkman, (1991)). Often, these components serendipitously have intuitive interpretations: For example, the first component usually represents “parallel” shifts in the curve, the second represents “twists,”; and the third represents “humping.”

There are, however, some drawbacks to principal components hedging. Most importantly, it requires the ability to compute the sensitivity of a fixed-income instrument or portfolio to changes in the components, a feature that most risk management systems lack. Additionally, it requires the ability to compute eigenvectors and eigenvalues, functions that are not readily available in spreadsheets or standard programming libraries.

The natural question arises: Does one method produce better hedges than the other? In this article, we seek to answer this question by putting both methods to the test. We compare their performance at hedging six bond portfolios over a one-and-half year period. We find that the two methods produce comparable hedges using one or two hedging instruments; when three hedging instruments are used, however, the severe instability of the third component degrades the three-instrument principal components hedge. This result is consistent with other findingsthat the third component tends to exhibit considerable instability over time when modeling yield curves (Wilson, 1994; Singh, 1995).

The Covariance Key-Rate Hedge

The covariance key rate hedge seeks the combination of hedging instruments that minimizes the variance in the portfolio. The steps of finding a covariance key rate hedge proceed as follows {see also Falkenstein and Hanweck (1996)}:

1) Choose a set of m key rates that adequately span the yield curve. These rates may be forward rates or zero rates. For example, for hedging a U.S. bond portfolio, a commonly used set of key rates are the nine zero rates: one, two, three, four five, seven, twenty, and thrity years.

2) Estimate the covariance matrix, V, of changes in the m key rates.[1]

3) Compute the m-vector of partial basis-point values (BPVs) for the portfolio, BPV(B). That is, $BPV_i(B)$ represents the change in the present value of the portfolio for a on1-basis-point change in the i-th key rate.

4) Compute the m x n-matrix of partial BPVs of the n hedging instruments, BPV(H). That is, $BPV_i(H_j)$ represents the change in the present value of hedging instrument j for a 1-basis-point change in the i-th key rate.

5) Solve for the n-vector of hedge ratios, β, that minimizes the variance of the hedged portfolio. The formula for β is

$$\beta = [BPV(H)' \; V \; BPV(H)]^{-1} \; [BPV(H)' \; BPV(B)] \tag{1}$$

Covariance key rate hedges can significantly reduce the standard deviation of a bond portfolio over traditional duration hedges and yield-beta hedges. We have found improvements of 25% on average over traditional duration hedging, and 13% over yield-beta methods.

The Principal-Components Hedge

Another hedging technique that has gained considerable interest in the fixed-income literature is hedging using *principal components* (Jones, 1991; Canabarro, 1995). One can think of a component as a fundamental factor that determines the movement of the yield curve. For example, the covariance key rate hedge uses the n key rates as the fundamental factors that determine changes in yields. At the other extreme, there are many one-factor models of the yield curve in which a single interest rate drives changes in the entire curve (e.g., Cox, Ingersoll, and Ross, 1985).

Factors do not have to be interest rates. A factor can just as easily be the curve itself (Ho and Lee, 1986; Heath, Jarrow, and Morton, 1992), its slope, or its curvature. The statistical method of principal components has proven to be a useful way of choosing factors that are mutually independent, explain a great deal of the variation in the yield curve, and often have sensible interpretations.

The procedure we use to find the principal components hedge is:

1) Choose a set of m key rates as for the covariance key rate hedge. Using k historical observations, compute the k x m matrix, X, of changes in these key rates.

2) Compute the eigenvectors and eigenvalues of the m x m matrix X'X. Each eigenvector is a *component*, and its eigenvalue indicates how much of the variation in the data that component explains. Sort the eigenvectors by their corresponding eigenvalues, largest to smallest, so the first component explains the most variance in the yield curve.[2]

3) Choose the number of hedging instruments, n, to use in the hedge.

4) Compute the n-vector of partial *component values*, CV(B), for each of the first n components (i.e., the n components with the largest eigenvalues). That is, $CV_i(B)$ represents the change in the present value of the portfolio for a unit change in the i-th component.

5) Compute the n x n matrix of partial CVs of the n hedging instruments, CV(H). That is, $CV_i(H_j)$ represents the change in the present value of hedging instrument j for a unit change in the i-th key rate.

6) Solve for the n-vector of hedge ratios, β, that is the principal-components hedge. The formula for β is

$$\beta = CV(H)^{-1}\, CV(B) \qquad (2)$$

Results

We compare covariance-consistent hedges to principal-components hedges for six bond portfolios: the on-the-run thirty-year, ten-year, and five-year U.S. Treasuries; a twenty-year Treasury (7.25% of May 15, 2016); a six-year Treasury (8% of May 15, 2001); and a zero-coupon Treasury strip (May 15, 2005). The hedging instruments are the two-year, five-year, ten-year, and thirty-year U.S. Treasury bond and note futures contracts. We use twelve key rates in our analysis: one- through ten-year, twenty-year, and thirty-year zero rates. These are bootstrapped from constant-maturity Treasury (CMT) rates from July 1994 through June 1996.

We begin the hedging tests on January 3, 1995. The key rate covariance matrix and the principal components were estimated from the key rate changes of the immediately preceding twenty-six weeks. Hedge ratios were computed as described above. Each week thereafter, we recorded the hedge error from the prior week, re-estimate the covariance matrix and components using the most recent twenty-six week period, and computed the new hedge ratios. This process resulted in seventy-seven weekly observations for each method and each portfolio.

We always use the hedging instruments whose duration is closest to that of the bond being hedged. We considered six hedges: one, two, and three-instrument principal components hedges (PC1-PC3); and one, two, and three-instrument covariance hedges (CC1-CC3).

Exhibit 1 shows the standard deviations of the hedged portfolio (i.e., standard deviation of hedge error) over the entire sample period. By this measure, for one- and two-instrument hedges (PC1/CC1 and PC2/CC2), the methods perform about equally well. Principal components outperforms covariance key rates when using one hedge instrument, but statistically significantly so only for the ten-year Treasury. When using two hedge instruments, covariance hedging performs somewhat better than principal components, although never statistically significantly so.

Adding a third principal component degrades hedge performance considerably compared with two principal components. Average hedge error increases from 0.203 to 0.304 (measured as the volatility of the hedged portfolio value as a percent of the notional amount of the bond). The decreased performance is most pronounced in the hedges of the ten- and thrity-year bonds and the ten-year Treasury strip. For the covariance key rate hedge, moving from using two hedging instruments to three futures instruments did not significantly change hedge performance.

We attribute the poor performance of the three-instrument PC hedge to the instability of the third component. Exhibit 2 shows the estimated PCs are over four twenty-six week periods. Note that the first two PCs relatively stable, while the third PC varies markedly, particularly in the third period. If one estimates the third PC using data from weeks 26 to50, and then uses this estimate to create a hedge for the subsequent six months, the third PC realized over the subsequent period is different enough to cause the three-PC hedge to underperform a hedge that simply ignores this component.

Note also the small absolute magnitude of the third PC (usually under 2 basis points). Since principal components are estimated with error, modest standard errors can make the third PC statistically indistinguishable from zero. In these cases, using the third PC provides nothing more than an example of over fitting one's data.

On the other hand, adding a third hedging instrument to the covariance key rate hedge does not significantly affect hedge performance {for reasons discussed in Falkenstein and Hanweck (1996)}. When using two hedge instruments, the "barbelling" of the portfolio with higher-and lower-duration hedging instruments helps to diversify the error in matching the performance of the underlying security.

Using three hedge instruments necessarily implies that two of them will have durations lower or higher than the underlying security. In this case, both of the instruments on one "side" of the underlying security perform a similar function, and their correlation with the underlying security and each other (relative to the hedge security on the other side) results in collinearity. This increases the standard errors in the estimated hedge ratios from the covariance key rate weighted-least squares regression. Therefore, the greater noise in the estimated hedge weights offsets the gains from adding more hedge instruments.

For example, to hedge the thirty-year on-the-run bond (so all hedging instruments have lower duration than the bond itself), a one-instrument covariance key rate hedge does about as well as either a two-instrument PC or covariance hedge.

Conclusion

Covariance key rate hedging is an effective and efficient method for constructing hedges of fixed-income portfolios. It compares favorably with principal components hedging methods on a variety of U.S. Treasury bonds and notes spanning the yield curve from five- to thrity-years.

One area where covariance keyrate hedging outperforms principal components is when using more than two hedging instruments. Adding the third instrument requires adding to the analysis the third component, which tends to be highly unstable over time. This instability adds variance to the principal components hedge, hurting its performance compared both to the covariance hedge and to principal-components hedges with fewer instruments.

The advantage of covariance key rate hedging is most apparent when hedging an entire portfolio of securities. In many fixe-income portfolios, the maturities of the bonds in the portfolio span several potential hedging points on the yield curve. Since this wide range of maturities introduces substantial risk form non-parallel shifts in interest rates, it is prudent to use a range of hedging ininstruments that match the range of maturities in the portfolio. This often necessitates more than two hedging instruments. using principal components with three or more hedging instruments (for example, the two-, five-, and ten-year Treasuries) appear to be an inferior choice, since the instability of the third component may in practice add variance to the portfolio rather than reduce it. covariance key rate hedging with three or more hedging instruments does not seem to suffer from this problem, so in this case it is a more robust hedging methodology.

When hedging a single security with one or two instruments, however, covariance key rate hedging and principal componenets give similar results. In this situation, other factors such as ease of implementation and transparency of the resulting hedges move to the forefron. In general, we believe that the robustness, simplicity, and clarity of covaraince key rate hedging make it an excellent tool for hedging fixed-income portfolios.

Exhibit 1. Standard deviations of hedge error over 77 weekly observations

in percentage of notional

	PC1	CC1	PC2	CC2	PC3	CC3	Unhedged**
30yr	0.345%	0.364	0.346	0.337	0.695*	0.354	1.597
20yr	0.292	0.301	0.282	0.284	0.293	0.285	1.406
10yr	0.235	0.287*	0.206	0.202	0.344*	0.208	1.085
6yr	0.137	0.150	0.128	0.128	0.126	0.129	0.836
5yr	0.096	0.100	0.096	0.094	0.094	0.095	0.715
Strip	0.188	0.222	0.160	0.155	0.271*	0.159	0.748
Average	0.215	0.237	0.203	0.200	0.304*	0.205	1.064

* Indicates hedging method produced more volatile hedge than its alternative at the 5% significance level (according to a standard F-test); e.g., CC1 hedge was more volatile than PC1 hedge for the 10yr at the 5% level.

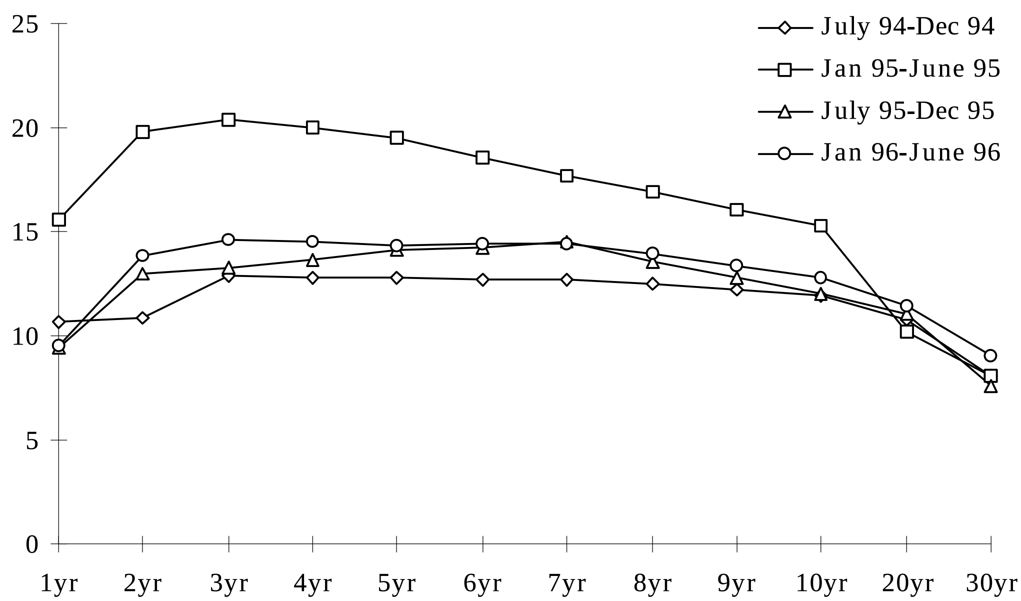
** Unhedged produced more volatility than the CC1 method at better than 1% level for all portfolios.

Exhibit 2. First, second, and third principal components

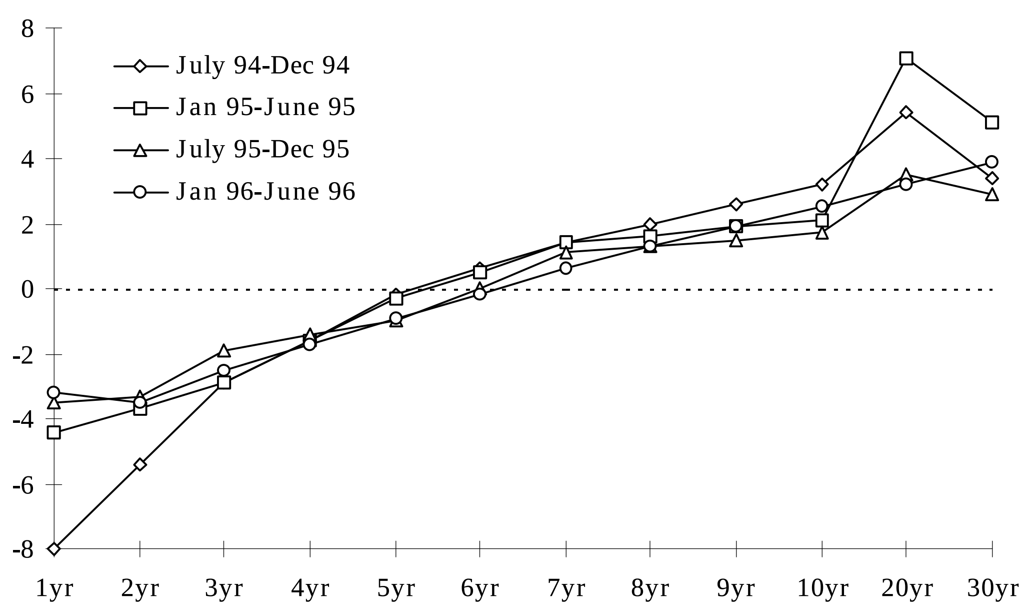
in basis points of zero-coupon yield, normalized to one standard deviation per week

estimated over stated six-month periods

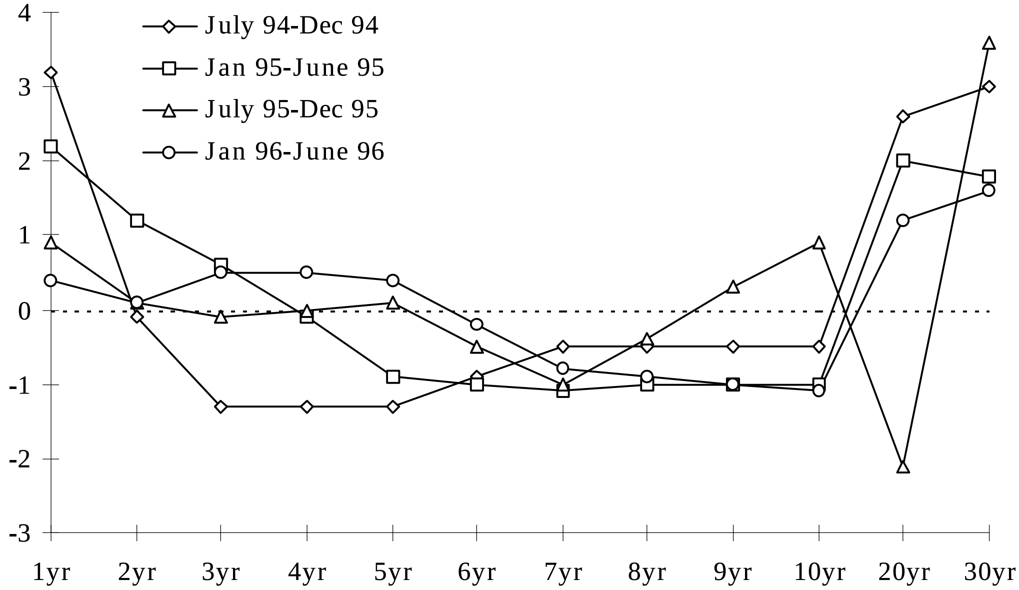
First component



Second component



Third component



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[1] One can straightforwardly estimate the covariance matrix from historical key-rate data. Another approach is to estimate the *correlation* matrix from historical data, but use implied volatilities from the options markets to get the standard deviations of the key rates. From the correlation matrix R and the diagonal matrix of standard deviations, S, the covariance matrix is simply V = S R S. This approach has the advantage that it uses contemporary, forward-looking volatilities from the options markets, and relies on historical data only for the correlation matrix, which tends to be more stable over time than the covariance matrix.

[2] Typically, each component is scaled by multiplying by the square root of the ratio of its eigenvalue and the number of observations, k. This allows one to interpret the component as a one-standard-deviation move per unit time interval (e.g., if observations are daily, the component would represent a 1-s.d. move per day).