# Downside Correlation and Expected Stock Returns\*

Andrew Ang
Columbia University and NBER

Joseph Chen
University of Southern California

Yuhang Xing
Columbia University

This Version: 12 Mar, 2002

JEL Classification: C12, C15, C32, G12

Keywords: asymmetric risk, cross-sectional asset pricing, downside correlation, downside risk, momentum effect

\*This paper was previously circulated under the title "Downside Risk and the Momentum Effect." The authors thank Brad Barber, Geert Bekaert, Alon Brav, John Cochrane, Randy Cohen, Kent Daniel, Bob Dittmar, Rob Engle, Cam Harvey, David Hirschleifer, Qing Li, Terence Lim, Toby Moskowitz, Akhtar Siddique, Bob Stambaugh and Zhenyu Wang. We especially thank Bob Hodrick for detailed comments. We thank seminar participants at Columbia University, NYU, USC, the Five Star Conference, and the NBER Asset Pricing Meeting for helpful comments. The authors acknowledge funding from a Q-Group research grant. Andrew Ang: aa610@columbia.edu; Joe Chen: joe.chen@marshall.usc.edu; Yuhang Xing: yx35@columbia.edu.

#### **Abstract**

If investors are more averse to the risk of losses on the downside than of gains on the upside, investors ought to demand greater compensation for holding stocks with greater downside risk. Downside correlations better capture the asymmetric nature of risk than downside betas, since conditional betas exhibit little asymmetry across falling and rising markets. We find that stocks with high downside correlations with the market, which are correlations over periods when excess market returns are below the mean, have high expected returns. Controlling for the market beta, the size effect, and the book-to-market effect, the expected return on a portfolio of stocks with the greatest downside correlations exceeds the expected return on a portfolio of stocks with the least downside correlations by 6.55% per annum. We find that part of the profitability of investing in momentum strategies can be explained as compensation for bearing high exposure to downside risk.

## 1 Introduction

According to the Capital Asset Pricing Model (CAPM), a stock's expected excess return is proportional to its market beta, which is constant across down-markets and up-markets. While this model has been rejected, modern factor models, like the Fama and French (1993) three-factor model, continue to maintain the symmetric nature of factor loadings across down-markets and up-markets. However, as early as Markowitz (1959), economists have realized that investors care differently about downside risk, than they care about total market risk. Markowitz advises constructing portfolios based on semi-variances, rather than on variances, since semi-variances weight upside risk (gains) differently from downside risk (losses). More recently, in Kahneman and Tversky (1979)'s loss aversion utility and in Gul (1991)'s first-order risk aversion utility, losses are weighted more heavily than gains in an investor's utility function. If investors dislike downside risk, they ought to demand higher compensations – in the form of higher expected returns – for holding assets with greater downside risk.

One natural extension of the CAPM, that takes into account this asymmetric treatment of risk, is the use of downside and upside betas (Bawa and Lindenberg, 1977). These downside betas are the market betas computed over periods for which the market return is below its mean (downside periods). However, downside betas produce little variations in the cross-section of expected returns in the data, since they are affected by the changes in idiosyncratic volatility and in market volatility across downside and upside periods. In particular, many authors, including Campbell et al. (2001), find that market volatility increases in down-markets and recessions. Moreover, Duffee (1995) finds that idiosyncratic volatility decreases in down-markets. Both of these effects cause conditional beta to have little asymmetry across the downside and the upside. In contrast, conditional correlations are immune from different volatility effects across upmarkets and down-markets, and exhibit significant asymmetries across downside versus upside moves by the market (Ang and Chen, 2001). This suggests that conditional correlations may be better able to capture the asymmetric nature of risk than conditional betas.

We find that stocks with high downside correlations, which we measure as highly correlated movements with the aggregate market in periods when markets fall, provide high expected returns. The portfolio of greatest downside correlation stocks outperforms the portfolio of lowest downside correlation stocks by 4.91% per annum. We show that downside correlations are not linked to low liquidity in down markets nor mechanically linked to past returns. After controlling for the market beta, the size effect, and the book-to-market effect, the greatest downside correlation portfolio outperforms the lowest downside correlation portfolio by 6.55% per annum.

Our research design follows the custom of constructing and adding factors to explain deviations from the Capital Asset Pricing Model (CAPM). While this approach does not speak to the nature of the risk premia, our goal is not to present a theoretical model that explains how downside risk is priced in equilibrium. Our goal is to demonstrate that a part of the factor structure in stock returns reflects variations in downside risk, measured by downside correlations. Not surprisingly, we find that while the Fama-French (1993) three-factor model cannot explain the variations in expected returns of stocks sorted by downside correlations, a factor reflecting the spread in expected returns induced by downside correlations explains these variations. We term this factor 'CMC', and find that it also helps to forecast economic downturns.

As an application of the CMC factor, we link the profitability of the Jegadeesh and Titman (1993) momentum strategies to downside risk. Existing explanations of the momentum effect are largely behavioral in nature and use models with imperfect formation and updating of investors' expectations in response to new information (Barberis, Shleifer and Vishny, 1998; Daniel, Hirshleifer and Subrahmanyam, 1998; Hong and Stein, 1999). These explanations rely on the assumption that arbitrage is limited, so that arbitrageurs cannot eliminate the apparent profitability of momentum strategies. Mispricing may persist because arbitrageurs need to bear some undiversifiable risk, and risk-averse arbitrageurs demand compensation for accepting such risk (Hirshleifer, 2001). We argue that these momentum strategies have high exposures to a systematic downside correlation factor. The intuition behind this story is that past winner stocks have high returns, in part, because during periods when the market experiences downside moves, winner stocks move down more with the market than past loser stocks.

The momentum portfolios load positively and significantly on the downside correlation factor. In particular, a linear two-factor model with the market and the CMC factor explains some of the cross-sectional variations among momentum portfolio returns. The downside correlation factor commands a significantly positive risk premium in cross-sectional tests, and retains its statistical significance in the presence of the Fama-French and momentum factors. However, the downside correlation factor only modestly reduces the Carhart (1997) WML momentum factor premium by 2.18% per annum, and hypothesis tests reject that this factor can fully account for the momentum effect.

<sup>&</sup>lt;sup>1</sup> Other authors use factors which reflect the size and the book-to-market effects (Fama and French, 1993 and 1996), macroeconomic factors (Chen, Roll and Ross, 1986), production factors (Cochrane, 1996), labor income (Jagannathan and Wang, 1996), market microstructure factors like volume (Gervais, Kaniel and Mingelgrin, 2001) or liquidity (Pástor and Stambaugh, 2001), and factors motivated from corporate finance theory (Lamont, Polk and Saá-Requejo, 2001).

Our findings are closely related to other studies which use factor models to account for the high momentum returns. Harvey and Siddique (2000) demonstrate that skewness is priced, and show that momentum strategies are negatively skewed. Unlike skewness or other centered moments, our conditional correlation measure emphasizes the asymmetry of risk across downside and upside market moves. Our findings are also related to DeBondt and Thaler (1987) who find that past winner stocks have greater downside betas than upside betas. We find that the spreads in expected returns from downside beta are very weak since conditional betas are roughly constant across upside and downside periods. In contrast, downside correlation portfolios produce large cross-sectional variations in expected returns.

The rest of this paper is organized as follows. Section 2 investigates the relation between higher-order moments and expected returns. We show that portfolios sorted by increasing downside correlations have increasing expected returns. On the other hand, portfolios sorted by other higher moments do not produce any discernable pattern in their expected returns. Section 3 explores if the patterns across portfolios of downside correlations are robust after controlling for some known effects. Section 4 details the construction of our downside correlation factor and shows that it commands an economically significant risk premium. We apply the downside correlation factor to price the momentum portfolios in Section 5. Section 6 concludes.

# 2 Higher-Order Moments and Expected Returns

We start with the relations between centered moments and expected returns in Section 2.1. Since this framework fails to produce significant spreads in expected returns, we turn our attention to downside and upside betas advocated by Bawa and Lindenberg (1977) in Section 2.2. High downside beta stocks have only slightly higher expected returns than low downside beta stocks. Section 2.3 examines the cause of this failure, and shows that the effect of changing idiosyncratic and market volatilities across downside and upside periods, masks the asymmetry of conditional betas across the downside and the upside. On the other hand, downside correlation is not affected by changing volatility effects, and exhibits highly asymmetric patterns across the downside and upside periods. Portfolio sorts based on downside correlations produce large spreads in expected returns, which we demonstrate in Section 2.4.

## 2.1 Centered versus Conditional Moments

Economic theory predicts that the expected return of an asset is linked to higher-order moments of the asset's return through the preferences of a marginal investor. The standard Euler equation

in an arbitrage-free economy is:

$$E_t[m_{t+1}r_{i,t+1}] = 0, (1)$$

where  $m_{t+1}$  is the pricing kernel or the stochastic discount factor and  $r_{i,t+1}$  is the excess return on asset i. If we assume that consumption and wealth are equivalent, then the pricing kernel is the marginal rate of substitution for the marginal investor:  $m_{t+1} = U'(W_{t+1})/U'(W_t)$ . By taking a Taylor expansion of the marginal investor's utility function, U, we can write:

$$m_{t+1} = 1 + \frac{W_t U''}{U'} MK T_{t+1} + \frac{W_t^2 U'''}{2U'} MK T_{t+1}^2 + \dots,$$
 (2)

where  $MKT_{t+1}$  is the rate of return on the market portfolio, in excess of the risk-free rate.

The coefficient on  $MKT_{t+1}$  in equation (2),  $W_tU''/U'$ , corresponds to the relative risk aversion of the marginal investor. The coefficient on  $MKT_{t+1}^2$  is studied by Kraus and Litzenberger (1976) and motivates Harvey and Siddique (2000)'s coskewness measure, where risk-averse investors prefer positively skewed assets to negatively skewed assets. Dittmar (2001) examines the cokurtosis coefficient on  $MKT_{t+1}^3$  and argues that investors with decreasing absolute prudence dislike cokurtosis.

If the systematic component of skewness or kurtosis are priced, then stocks sorted by coskewness or cokurtosis should exhibit cross-sectional spreads in expected returns. When stocks are sorted into decile portfolios by increasing past coskewness, we do find that stocks with more negative coskewness have higher returns. However, the difference between the portfolio of stocks with the most negative coskewness and the portfolio of stocks with the most positive coskewness is only 1.79% per annum, which is not statistically significant at the 5% level (t-stat = 1.17). When we sort on cokurtosis, high cokurtosis stocks have slightly lower expected returns than low cokurtosis stocks, which is opposite of that predicted by theory.

Why might centered moments like coskewness and cokurtosis fail to pick up much pattern in the cross-section of stock returns? If the marginal investor's utility is kinked, skewness and other centered moments may not effectively capture the asymmetry in aversion to risk across upside and downside moves. Empirical tests reject standard specifications for U, such as power utility, and leave unanswered what the most appropriate representation for U is. However, economic theory does not restrict the utility function U to be smooth. For instance, Kahneman and Tversky (1979)'s loss aversion utility function and Gul (1991)'s first-order risk aversion utility function have a kink at the reference point to which an investor compares gains and losses. These asymmetric, kinked utility functions suggest that polynomial expansions of U, such as the expansion used by Bansal, Hsieh and Viswanathan (1993), may not be a good global approximations of U. In particular, standard polynomial expansions may not capture

risk which differs across down or up markets. In an effort to capture the asymmetric nature of risk, we turn to moments conditioned on downside and upside moves of the market.

#### 2.2 Downside and Upside Betas

A natural starting point for examining asymmetries in risk is to consider downside and upside betas. Following Bawa and Lindenberg (1977), we define downside beta  $\beta^-$  and upside beta  $\beta^+$  as:

$$\beta^{-}(\theta) = \frac{\operatorname{cov}(r_{i,t}, MKT_{t}|MKT_{t} < \theta)}{\operatorname{var}(MKT_{t}|MKT_{t} < \theta)}$$
and 
$$\beta^{+}(\theta) = \frac{\operatorname{cov}(r_{i,t}, MKT_{t}|MKT_{t} > \theta)}{\operatorname{var}(MKT_{t}|MKT_{t} > \theta)},$$
(3)

where  $r_{i,t}$  is the excess stock return and  $MKT_t$  is the excess market return. The parameter  $\theta$  is a conditioning level. Hence,  $\beta^-(\theta)$  is the beta among observations where the market return is less than  $\theta$ , and  $\beta^+(\theta)$  is the beta among observations where the market return is greater than  $\theta$ . In the case where  $\theta = \overline{MKT}$ , where  $\overline{MKT}$  is the mean excess market return, we abbreviate the notation to  $\beta^-(\theta = \overline{MKT}) = \beta^-$  and  $\beta^+(\theta = \overline{MKT}) = \beta^+$ .

Downside and upside betas capture the notion of asymmetric exposures to risk across periods when the market falls and periods when the market rises. These moments are different from centered moments because they emphasize the asymmetry across upside market moves and downside market moves explicitly by the conditioning level  $\theta$ . Computing  $\beta^-(\theta)$  and  $\beta^+(\theta)$  is simple: we take those observations which satisfy the conditioning requirement based on  $\theta$ , and then compute the betas on this subsample of observations. If the agents in the economy are more sensitive to losses than they are to gains, then stocks with greater downside risk should provide greater compensation.

To examine if downside or upside betas are related to expected returns, we sort stocks based on their downside betas and on their upside betas to form portfolios based on these sorts. Since we have fewer observations available to us as  $\theta$  moves further away from the mean, we focus on the conditioning level of  $\theta = \overline{MKT}$ , so that roughly half of the observations are used to compute  $\beta^-$  and  $\beta^+$  for each stock. We rank stocks into deciles, and calculate the value-weighted holding period return of the portfolio of stocks in each decile over the following month. We rebalance these portfolios each month. Appendix A provides further detail on portfolio construction.

Table (1) presents the characteristics of portfolios formed by the sorts of stocks on their downside betas and on their upside betas, as well as on their unconditional betas. Panel A of

Table (1) shows the summary statistics of stocks sorted by the unconditional betas. The column labeled ' $\beta$ ' shows the unconditional betas of each portfolios calculated at the monthly frequency over the whole sample. This column shows that the portfolios constructed by ranking stocks on past unconditional betas retain their beta-rankings in the post-formation period. However, confirming many previous studies, Panel A shows that there is no pattern in the expected returns of the beta-sorted portfolios.

Panel B of Table (1) reports the summary statistics of stocks sorted by downside beta,  $\beta^-$ . The columns labeled ' $\beta$ ' and ' $\beta^-$ ' list the post-formation unconditional betas and downside betas, respectively. Portfolios with higher past downside beta have higher unconditional betas. Sorting on past  $\beta^-$  also produces large ex-post formation spreads in downside beta, that is, downside beta is also persistent. There is a weakly increasing, but mostly humped-shaped pattern in the mean returns of the  $\beta^-$  portfolios. However, the difference in the returns is not statistically significant. In Panel C, stocks sorted on past  $\beta^+$  exhibit no spread in average returns. Hence, while there appears to be a weak spread in the expected returns across downside betas, upside beta does not seem to be priced.

#### 2.3 The Failure of Conditional Beta Measures

In this section, we investigate why the conditional beta measures fail to produce a significant relation between downside betas and expected returns. One reason why the effect of downside beta is weak is that there is little difference across downside and upside betas in the data, so the downside beta picks up very little asymmetry in risk.

Panel A of Figure (1) shows the average downside and upside beta for various conditioning levels on the x-axis across the 48 Fama-French (1997) industry portfolios at the monthly frequency. On the LHS of the x-axis for  $x \leq 0$ , the figure displays the average  $\beta^-(\theta)$  across all 48 industry portfolios, where  $\theta$  is x standard deviations below the unconditional mean of the market. For example, at x = -1, the figure plots  $\beta^-(\theta = \overline{MKT} - SE_{MKT})$ , where  $\overline{MKT}$  is the unconditional mean of the excess market return and  $SE_{MKT}$  is the unconditional volatility of the excess market return. At x = 0, the figure plots  $\beta^-(\theta = \overline{MKT})$ . Similarly, on the RHS of the x-axis for  $x \geq 0$ , the figure displays  $\beta^+(\theta)$ , for  $\theta$  representing x standard deviations above the mean of the market. There are two points plotted at x = 0 representing  $\beta^-(\theta = \overline{MKT}) \equiv \beta^-$  and  $\beta^+(\theta = \overline{MKT}) \equiv \beta^+$ . To construct the average industry  $\beta^-(\theta)$ , we first select the sample of observations which satisfies the conditioning requirement based on  $\theta$ . Then, the individual  $\beta^-(\theta)$  for each industry is computed for each sample. The figure graphs the average  $\beta^-(\theta)$  across the industries for each  $\theta$ . The procedure is repeated for the average

 $\beta^+(\theta)$  across the 48 industries.

In Panel A of Figure (1), the average  $\beta^-$  across the 48 industries is only slightly higher (4.8%) than the average  $\beta^+$  at  $\theta = \overline{MKT}$  (x = 0). As we condition on more extreme market moves (as  $\theta$  becomes larger in absolute value), the plot shows little difference between  $\beta^-(\theta)$  and  $\beta^+(\theta)$ . When we examine the differences between  $\beta^-$  and  $\beta^+$  for the industries individually, we see the reason why. Panel B shows the ratio of  $\beta^-$  to  $\beta^+$  across each of the 48 industries at  $\theta = \overline{MKT}$ . The downside beta is greater than the upside beta for only 25 out of the 48 industries. In summary, there is little asymmetry in conditional betas across upside and downside movements of the market.

To further investigate the failure of the conditional betas, we decompose the downside and upside betas into a conditional correlation term and a ratio of conditional total volatility to conditional market volatility:

$$\beta^{-}(\theta) = \frac{\operatorname{cov}(r_{i,t}, MKT_{t}|MKT_{t} < \theta)}{\operatorname{var}(MKT_{t}|MKT_{t} < \theta)} = \rho^{-}(\theta) \times k^{-}(\theta)$$
and 
$$\beta^{+}(\theta) = \frac{\operatorname{cov}(r_{i,t}, MKT_{t}|MKT_{t} > \theta)}{\operatorname{var}(MKT_{t}|MKT_{t} > \theta)} = \rho^{+}(\theta) \times k^{+}(\theta),$$
(4)

where downside and upside correlation  $(\rho^-(\theta))$  and  $\rho^+(\theta)$ , respectively) are given by:

$$\rho^{-}(\theta) = \operatorname{corr}(r_{i,t}, MKT_t | MKT_t < \theta)$$
and 
$$\rho^{+}(\theta) = \operatorname{corr}(r_{i,t}, MKT_t | MKT_t > \theta),$$
(5)

and  $k^-(\theta)$  and  $k^+(\theta)$  are ratios of conditional volatilities:

$$k^{-}(\theta) = \frac{\sigma(r_{i,t}|MKT_{t} < \theta)}{\sigma(MKT_{t}|MKT_{t} < \theta)}$$
and 
$$k^{+}(\theta) = \frac{\sigma(r_{i,t}|MKT_{t} > \theta)}{\sigma(MKT_{t}|MKT_{t} > \theta)}.$$
(6)

As with the notation for downside and upside beta, when  $\theta = \overline{MKT}$ , we abbreviate to  $\rho^-(\theta = \overline{MKT}) \equiv \rho^-$ ,  $\rho^+(\theta = \overline{MKT}) \equiv \rho^+$ ,  $k^-(\theta = \overline{MKT}) \equiv k^-$ , and  $k^+(\theta = \overline{MKT}) \equiv k^+$ .

The asymmetry in conditional correlations is much stronger than the asymmetry in betas across market downside and upside movements. Panel C of Figure (1) looks at the effects of downside and upside correlations across various  $\theta$ . There is a marked asymmetry across the average downside and upside correlations for the 48 industry portfolios, with a sharp break at  $\theta = \overline{MKT}$  (x = 0). Ang and Chen (2001) show that if returns are drawn from a normal distribution, as  $\theta$  becomes larger in absolute magnitude, the downside and upside correlations must be symmetric and tend to zero. While upside conditional correlations decrease as  $\theta$ 

increases, downside correlations do not decrease as  $\theta$  decreases. Panel D of Figure (1) shows that at  $\theta = \overline{MKT}$ , the point estimates of the downside correlations,  $\rho^-$ , are higher than the upside correlations,  $\rho^+$ , for every industry portfolio.

The second term in equation (4) is the reason why there is an asymmetry in conditional correlations but not in conditional betas. The terms,  $k^-(\theta)$  and  $k^+(\theta)$ , are the ratios of total asset volatility to market volatility, conditional on downside and upside market moves. We plot these volatility ratios in Panel E of Figure (1). Downside volatility ratios are much lower than volatility ratios on the upside. The corresponding Panel F shows the ratio  $k^-/k^+$  for each industry. In all but one of 48 industries, the downside volatility ratio is higher than the upside volatility ratio.

There are two effects that explain why on average  $k^-(\theta) < k^+(\theta)$ . First, the denominator of  $k^-(\theta)$  and  $k^+(\theta)$  in equation (6) is market volatility. Market volatility is asymmetric and higher after negative shocks to expected returns. Hence, conditional on the downside, the denominator of equation (6) is larger for  $k^-$  than for  $k^+$ . Second, Duffee (1995) finds that cross-sectional dispersion is asymmetric and idiosyncratic volatility decreases when the stock market falls. This makes the ratio of total volatility to market volatility larger for  $k^+$  than for  $k^-$ . Both of these effects contribute to downside  $k^-(\theta)$  being lower than  $k^+(\theta)$  on average, as Panels E and F of Figure (1) demonstrate.

The decrease of  $k^-(\theta)$  on the downside relative to  $k^+(\theta)$  on the upside counter-acts the increase of  $\rho^-(\theta)$  on the downside relative to  $\rho^+(\theta)$  on the upside. Multiplying the points in the conditional correlation line in Panel C together with the corresponding point in the conditional volatility ratio line in Panel E produces a relatively flat effect for the conditional betas in Panel A. Hence, the conditional beta is relatively flat across downside and upside movements because the increase in correlations on the downside is muted by the decrease in conditional idiosyncratic volatility and by the increase in market volatility.

## 2.4 Downside Correlations and Expected Returns

While the cross-sectional spreads of expected returns of stocks sorted on downside and upside betas are small, we now demonstrate that sorting on the correlation component of the beta produces a large spread in expected returns, in particular for downside correlations. The conditional correlations are asymmetric over downside and upside movements, as opposed to the relatively low asymmetry in conditional betas across the downside and upside. Conditional correlations are unaffected by different idiosyncratic and market volatility across upside and downside moves, whereas these effects cause conditional beta to have little downside versus

upside asymmetry. Sorting on the other component of beta, the ratio of total to market volatility, produces no pattern in expected returns. Hence, conditional correlations may be better able to capture asymmetries in risk than conditional betas.

Table (2) lists monthly summary statistics of the portfolios sorted by  $\rho^-$  and  $\rho^+$ . We choose the same conditioning level,  $\theta = \overline{MKT}$ , as the sorts for the conditional betas in Table (1). Panel A of Table (2) contains the results for stocks sorted on past  $\rho^-$ . The first column lists the mean monthly holding period returns of each decile portfolio. Stocks with the highest past downside correlations have the highest returns. Going from the portfolio of lowest downside correlations (portfolio 1), to the portfolio of highest downside correlations (portfolio 10), the average return almost monotonically increases. The return differential between the portfolios of the highest decile  $\rho^-$  stocks and the lowest decile  $\rho^-$  stocks is 4.91% per annum (0.40% per month). This difference is statistically significant at the 5% level (t-stat = 2.26), using Newey-West (1987) standard errors with 3 lags.

The portfolio of stocks with the highest past downside correlations have the highest betas. Since the CAPM predicts that high beta assets have high expected returns, we investigate in Section 3 if the high returns of these portfolios are explained by the market betas. However, the high returns on these portfolios do not appear to be attributable to the size effect or the book-to-market effect. The columns labeled "Size" and "B/M" show that high  $\rho^-$  stocks tend to be large stocks and growth stocks. Size and book-to-market effects would predict high  $\rho^-$  stocks to have low returns rather than high returns. The  $\rho^-$  portfolios are also flat in leverage, so leverage also cannot be driving the pattern in expected returns.

We also control for the size and book-to-market effect using the Fama-French (1993) three-factor model. We take the time-series alphas from a regression of a  $\rho^-$  decile's excess portfolio returns onto MKT, SMB and HML factors:

$$r_{it} = a_i + b_i MKT_t + s_i SMB_t + h_i HML_t + \epsilon_{it}. \tag{7}$$

These alphas are reported in the column labeled 'FF  $\alpha$ ' of Table (2), and they maintain their nearly monotonic rankings. The difference in the Fama-French alphas between the decile 10 portfolio and the decile 1 portfolio is 0.53% per month, or 6.55% per annum with a p-value 0.00. Hence, the variation in downside risk in the  $\rho^-$  portfolios is not explained by the Fama-French model. In fact, controlling for the market, the size factor and the book-to-market factor increases the differences in the returns from 4.91% to 6.55% per annum.

The second to the last column calculates the post-formation conditional downside correlation of each decile portfolio. These post-formation period  $\rho^-$  are monotonically increasing, which indicates that the top decile portfolio, formed by taking stocks with the highest

conditional downside correlation over the past year at the daily frequency, is the portfolio with the highest downside correlation over the whole sample at the monthly frequency. This implies that using past  $\rho^-$  is a good predictor of future  $\rho^-$  and that downside correlations are persistent.

The last column lists the downside betas,  $\beta^-$ , of each decile portfolio. The  $\beta^-$  column shows that the  $\rho^-$  portfolios have a fairly flat  $\beta^-$  pattern. Hence, the spread in expected returns of the  $\rho^-$  portfolios is not due to  $\beta^-$ . We contrast this with the ex-post formation  $\rho^-$  of the  $\beta^-$  portfolios in Panel B of Table (1). The  $\beta^-$  portfolios have a slightly humped shape pattern (increasing and then decreasing) of expected returns. The  $\rho^-$  statistics of the  $\beta^-$  portfolios have the same humped shape pattern.

Panel B of Table (2) shows the summary statistics of stocks sorted by  $\rho^+$ . In contrast to the stocks sorted by  $\rho^-$ , there is no discernable pattern between the mean returns and upside correlations. However, the patterns in the  $\beta$ 's, market capitalizations, and book-to-market ratios of stocks sorted by  $\rho^+$  are similar to the patterns found in  $\rho^-$  sorts. In particular, high  $\rho^+$  stocks also tend to have higher betas, tend to be large stocks, and tend to be growth stocks. The last two columns list the post-formation  $\rho^+$  and  $\beta^+$  statistics. Here, both  $\rho^+$  and  $\beta^+$  increase monotonically from decile 1 to 10, but portfolio sorts by  $\rho^+$  do not produce any pattern in their expected returns.

In summary, Table (2) shows that assets with higher downside correlations have higher returns. The difference between the first and tenth decile of raw returns is 4.91% per annum, but this increases to 6.55% per annum controlling for the Fama-French (1993) factors. Downside beta does not pick up this spread in expected returns because it exhibits little asymmetry across downside and upside movements. In contrast to the strong relationship between expected returns and downside moments, expected returns do not seem to be related to upside conditional moments ( $\beta^+$  or  $\rho^+$ ).

# **3 Pricing the Downside Correlation Portfolios**

In this section we conduct a battery of tests to try to price the downside correlation effect. Section 3.1 examines if the expected returns on downside correlation portfolios can be explained by the market beta. Section 3.2 examines if these portfolio returns can be explained by the size effect, the book-to-market effect, or the momentum effect. Section 3.3 asks if the high downside correlation expected returns are robust across various subsamples. We show that downside correlation is not mechanically linked to past returns nor related to periods of low liquidity in Sections 3.4 and 3.5, respectively. Section 3.6 interprets our findings.

#### 3.1 Can Market Risk Price Downside Correlation?

In Table (2), the downside correlation portfolios display increasing betas with increasing  $\rho^-$ . This raises the concern that the expected returns on the downside correlation portfolios can be explained by the market beta. To allay such concerns, we work with twenty portfolios formed in the following manner. Stocks are first sorted into two groups (high beta versus low beta) according to their past betas over the past year at the daily frequency. Each group consists of one half of all firms. Then, within each beta group, we rank stocks based on their  $\rho^-$ , also computed using daily data over the past year into decile portfolios. This gives us  $2 (\beta) \times 10 (\rho^-)$  portfolios. When we average across the beta portfolios for each  $\rho^-$  decile, we find the spread in  $\rho^-$  controlling for the beta effect in ten decile portfolios. These decile beta-controlled  $\rho^-$  portfolios have near-flat uniform ex-post formation betas, which indicates that the double-sort is successful at controlling for beta.

The difference between the tenth and the first decile beta-controlled  $\rho^-$  portfolios is 0.56% per month, or 6.95% per annum, with a p-value of 0.00. A Gibbons, Ross and Shanken (1989) (GRS) F-test to jointly test if the portfolio alphas are zero rejects with a p-value of 0.00. We also consider Carhart (1997) four-factor model, which consists of the Fama-French factors plus a momentum factor, WML:

$$r_{it} = a_i + b_i MKT_t + s_i SMB_t + h_i HML_t + w_i WML_t + \epsilon_{it}.$$
(8)

Similar to the Fama-French time-series regressions, the alphas are still significant, with a difference of 0.44% per month, or 5.40% per annum, between the tenth and the first decile portfolios. A GRS test also rejects with a p-value of 0.00. Since the effect of downside correlation remains after controlling for the market beta, we conclude that downside correlation cannot be explained by market risk.

#### 3.2 Can Other Risk Factors Price Downside Correlation?

While we can control for the effect of market beta by forming additional double-sort portfolios, this strategy quickly becomes difficult to implement if we try to control for the effects of multiple factors. In this section, we run Fama-MacBeth (1973) cross-sectional tests (described in the Appendix) to test if multiple risk factors can price the spread in expected returns produced by downside correlation. In particular, we focus on the standard Fama-French (1993) model, and augment this with the Carhart (1997) WML momentum factor.

To run these tests, we work with the 20  $\rho^-$  portfolios described in the previous section to control for the beta effect. Table (3) reports the Fama-MacBeth cross-sectional estimates of the

Fama-French (1993) factor premiums:

$$E(r_{it}) = \lambda_0 + \lambda_{MKT}b_i + \lambda_{SMB}s_i + \lambda_{HML}h_i, \tag{9}$$

where  $\lambda_j$  is the premium of factor j. We also report the Carhart (1997) model factor premiums:

$$E(r_{it}) = \lambda_0 + \lambda_{MKT}b_i + \lambda_{SMB}s_i + \lambda_{HML}h_i + \lambda_{WML}w_i.$$
 (10)

In both cases, the factor premiums are all statistically insignificant and the size premium is estimated to be negative. These results are driven by the inability of these standard factors to price the  $\rho^-$  portfolios. We reject that the portfolio alphas are jointly zero using a GRS test.

In Figure (2), we plot the alphas and factor loadings from the Carhart four-factor model. Figure (2) orders the 20 portfolios so that portfolios 1-10 correspond to the low beta group, and portfolios 11-20 correspond to the high beta group. The portfolio alphas in the top panel reflect the spread in expected returns moving from low  $\rho^-$  to high  $\rho^-$ , but are much more pronounced in the high beta group. The factor loadings for MKT in the bottom panel reflect the low-high beta sorting, and are largely flat within each beta group. The factor loadings for SMB go the wrong way, so that high  $\rho^-$  firms with high returns have small SMB loadings. The HML factor also does not account for the  $\rho^-$  effect, since the HML factor loadings increase in the high beta group and are flat across the low beta group. Finally, the WML factor loadings are largely flat and very small.

## 3.3 Is the Downside Correlation Effect Stable Across Subsamples?

A simple robustness exercise is to check if the spread in the  $\rho^-$  correlation portfolios remains statistically significant in various subsamples. The top panel of Figure (2) also plots the alphas from the four-factor model from the full sample (Jan 1964 to Dec 1999), from Jan 1964 to Dec 1981 and from Jan 1982 to Dec 1999. The alphas have almost the same increasing pattern in each subsample. Hence, the same qualitative relationship between  $\rho^-$  and expected returns holds in different subperiods.

To conduct a more formal test for stability over different subsamples, we compute the difference between the alphas of the tenth and the first decile beta-controlled  $\rho^-$  portfolios, which are reported in Table (4). The difference in the alphas between portfolio 10 and 1 are statistically significant at the 1% level for both the three-factor and the four-factor models across the whole sample period. For the Fama-French model alphas in the first column, the alphas are still large and statistically significant when the sample is split into two separate calendar periods, and remain significant when the sample is split into NBER expansions and recessions.

In the last two columns of Table (4) we report the alphas from the four-factor model. These alphas are slightly smaller than the alphas from the Fama-French model and are also highly significant across NBER expansions and recessions. However, when the sample is split into two calendar periods, the difference in the alphas is near-significant (p-value = 0.06) over Jan 1964 - Dec 1981, but is highly statistically significant over the second period (Jan 1982 - Dec 1999). Nevertheless, the alphas are still of a large magnitude across various subsamples. A more serious concern is that since the alphas including the WML factor are smaller than the alphas from the Fama-French model, this raises the question that a large part of the high expected returns induced by high downside correlations may be due to momentum.

#### 3.4 Is Downside Correlation Capturing Past Returns?

There are several similiarities between the Jegadeesh-Titman (1993) momentum effect and downside risk, which raises the concern that downside correlation is merely a noisy measure of past returns. First, like momentum, the downside correlation alphas are exacerbated by size and value effects (Fama and French, 1996; Grundy and Martin, 2001). Second, controlling for momentum in the time-series regressions reduces the alphas of the downside correlation portfolios. We show in this section that downside correlations are not mechanically linked to past returns, hence the momentum effect.

To disentangle the effects of past returns and downside correlations, we perform a double  $5\times 5$  sort across past 6 months returns and downside correlations. At each month, we first sort all stocks into quintiles based on their past 6 month returns. Then to control for past returns, we sort stocks within each past return quintiles into additional quintiles based on  $\rho^-$ . This procedure creates 25 portfolios, and we take the averages of the  $\rho^-$  portfolios across past return quintiles.

We report the alphas from the Fama-French three-factor model of these five portfolios in Panel A of Table (5). Controlling for past returns, these averages of downside correlation portfolios show cross-sectional dispersion in  $\rho^-$ . Their alphas are statistically significant, and the difference between the first and fifth portfolio alphas is 0.33% per month, which is also significant with a p-value = 0.00. In Table (4), controlling for momentum over the first subsample period (Jan 1964 - Dec 1981) yielded only a borderline significant result. Now, when we control for the momentum effect by the double portfolio sort, we find that the difference in the alphas becomes highly significant (t-stat = 2.50) over this first subsample period (Jan 1964 - Dec 1981). Hence, after controlling for momentum, high downside correlation stocks still have high returns.

#### 3.5 Is Downside Correlation Liquidity?

A number of studies find that liquidity of the market dries up during down markets. Pástor and Stambaugh (2001) construct an aggregate liquidity measure which uses signed order flow, and find that their liquidity measure spikes downwards during periods of extreme downward moves, such as during the October 1987 crash, and during the OPEC oil crisis. Jones (2001) also find that the bid-ask spreads increase with market downturns, while Chordia, Roll and Subrahmanyam (2000) find a positive association at a daily frequency between market-wide liquidity and market returns. These down markets, which seem to be correlated with systematically low liquidity, are precisely the periods which downside risk-averse investors dislike.

To study the relation between downside risk and liquidity, we follow Pástor and Stambaugh (2001) and reconstruct their aggregate liquidity measure, L, detailed in Appendix A. After constructing the liquidity measure, we assign a historical liquidity beta,  $\beta^L$ , at each month, for each stock listed on NYSE, AMEX and NASDAQ. This is done using monthly data over the previous 5 years from the following regression:

$$r_{it} = a_i + \beta^L L_t + b_i MKT_t + s_i SMB_t + h_i HML_t + \epsilon_{it}, \tag{11}$$

where  $L_t$  is the aggregate liquidity measure.

Since each stock i in our sample has a downside correlation ( $\rho_{i,t}^-$ ) and a liquidity beta ( $\beta_{i,t}^L$ ) for each month, we can examine the unconditional relations between the two measures. First, we compute the cross-sectional correlation between  $\rho_{i,t}^-$  and  $\beta_{i,t}^L$  at each time t, and then average over time to obtain the average cross-sectional correlation between downside risk and liquidity. The average cross-sectional correlation is -0.0108, which is close to zero. We obtain the average time-series correlation between  $\rho_{i,t}^-$  and  $\beta_{i,t}^L$  by computing the correlation between these two variables for each firm across time, and then averaging across firms. The average time-series correlation is -0.0029, which is also almost zero. Hence, our measure of downside risk is almost orthogonal to Pástor and Stambaugh's measure of aggregate liquidity risk.

To further investigate the relation between downside risk and aggregate liquidity, we perform a  $5 \times 5$  double sort based on liquidity and  $\rho^-$ . At each month, we independently sort stocks into two quintile groups based on  $\beta^L$  and  $\rho^-$ . The intersection of these two quintile groups forms 25 portfolios sorted by  $\beta^L$  and  $\rho^-$ . We take the average of the  $\rho^-$  portfolios across  $\beta^L$  quintiles, and report the intercept coefficients from a Fama-French (1993) factor time-series regression of these average portfolios in Panel B of Table (5).

We observe a similar pattern in the average returns moving from low  $\rho^-$  to high  $\rho^-$  portfolios as in Table (2). There is negative mispricing in the low  $\rho^-$  portfolios and positive mispricing

in the high  $\rho^-$  portfolio. The difference between  $a_5$  and  $a_1$  is 0.26% per month, which is statistically significant at the 1% level. Hence, even after controlling for liquidity risk using the Pástor-Stambaugh liquidity measure, there remains significant mispricing of downside risk relative to the Fama-French three factor model.

#### 3.6 Is Downside Correlation Risk?

Since the mean-variance framework is rejected by various asset pricing tests, it is not surprising that higher-order moments play a role in explaining cross-sectional variations in expected stock returns. However, which higher-order moments are important for cross-sectional pricing is still a subject of debate. We have shown that portfolios sorted by downside correlations produce large spreads in expected returns which cannot be explained by the market beta, the Fama-French (1993) SMB and HML factors, by momentum, or by liquidity. The spread in returns is also robust across subsamples.

One puzzling result is why high downside correlation stocks exhibit significantly high variations in expected returns, while the difference in expected returns between stocks with high downside beta versus low downside beta is weak. Downside correlation is scaled to emphasize comovements in only direction, while downside beta measures both magnitude and direction. We acknowledge that it is hard to think of a model where the magnitude does not matter but only the direction does. Even in models with one-sided constraints, for example binding short-sales constraints (Chen, Hong and Stein, 2001) or wealth constraints (Kyle and Xiong, 2001), there should be both direction and magnitude effects.

However, downside correlation is the component of downside beta which is unaffected by changes in idiosyncratic and market volatilities across downside and upside movements. Conditional correlations strongly differ across up and down markets. However, in down markets, idiosyncratic stock volatility decreases while market volatility increases (Duffee, 1995). This means that the ratio of total volatility to market volatility decreases in down markets, which causes the betas to have little variations across downside or upside moves of the market. Conditional correlations are unaffected by changing idiosyncratic and market volatilities and they are much more asymmetric across market downside and upside periods. Hence, conditional correlations are a good statistic to measure asymmetries in risk.

Lacking an economic model, we are reluctant to say that the high expected returns commanded by high downside correlation portfolios are due to risk. However, the standard risk factors cannot price, or even exacerbate, the expected returns of the downside correlation portfolios. In order to summarize this effect in a model, we capture the spread in returns produced by downside correlations by constructing a factor that mimicks this downside correlation effect. This factor should be able to price the downside correlation portfolios (by construction) and may also help explain other variations in the cross-section of expected stock returns.

#### 4 A Downside Correlation Factor

In this section, we build a factor that reflects the high expected returns earned by stocks with high downside correlations. We describe the construction of this factor in Section 4.1, and show that it prices the downside correlation portfolios in Section 4.2. Section 4.3 shows that the downside correlation factor significantly predicts economic recessions.

## **4.1** Constructing the Downside Correlation Factor

We construct a downside correlation factor that captures the return premium between stocks with high downside correlations and stocks with low downside correlations, which we call the CMC factor for "high Correlations Minus low Correlations". The CMC factor goes long stocks with high downside correlations, which have high expected returns, and shorts stocks with low downside correlations, which have low expected returns.

In constructing the CMC factor, we are careful to control for the positive relation shown in Table (2) between the beta and the downside correlation. The CMC factor extracts the spread in expected returns due to downside correlation, controlling for the beta. Each month, we place half of the stocks based on their  $\beta$ 's into a low  $\beta$  group and the other half into a high  $\beta$  group. Then, within each  $\beta$  group, we rank stocks based on their  $\rho^-$  into three groups: a low, a medium and a high  $\rho^-$  groups with the cutoffs at 33.3% and 66.7%. This sorting procedure creates six portfolios in total.

We calculate monthly value-weighted returns for each of these 6 portfolios. Within the low  $\beta$  group, the portfolio returns increase from the low  $\rho^-$  portfolio to the high  $\rho^-$  portfolio, with an annualized difference of 2.40% (0.20% per month). Moving across the low  $\beta$  group, portfolio returns of the  $\rho^-$  portfolios increase, while the beta remains flat at around  $\beta=0.66$ . The return also increases with increasing  $\rho^-$  within the high  $\beta$  group. Within the high  $\beta$  group, the difference in returns of the high  $\rho^-$  and low  $\rho^-$  portfolios is 3.24% per annum (0.27% per month), with a t-statistic of 1.98, but the  $\beta$  decreases with increasing  $\rho^-$ . Therefore, the higher returns associated with portfolios with high  $\rho^-$  are not rewards for bearing higher market risk, but are rewards for bearing higher downside correlation.

For each  $\rho^-$  group, we take the simple average across the two  $\beta$  groups and create three portfolios, which we call the  $\beta$ -balanced  $\rho^-$  portfolios. Moving across the  $\beta$ -balanced  $\rho^-$  portfolios, mean returns monotonically increase with  $\rho^-$ . This increase is accompanied by a monotonic decrease, rather than an increase, in beta. We define our downside risk factor, CMC, as the return on a zero-cost strategy of going long the  $\beta$ -balanced high  $\rho^-$  portfolio and shorting the  $\beta$ -balanced low  $\rho^-$  portfolio. This strategy is rebalanced monthly. The return on this strategy is 2.80% per annum (0.23% per month) with a t-statistic of 2.35 and a p-value of 0.02.

Since we include every firm listed on NYSE/AMEX and NASDAQ, and use daily data, the impact of small illiquid firms might be a concern. We address this issue in two ways. First, all of our portfolios are value-weighted, which reduces the influence of smaller firms. Second, we perform the same sorting procedure as above, but exclude firms that are smaller than the tenth NYSE percentile. With this alternative procedure, we find that CMC is still statistically significant with an average monthly return of 0.23% and a t-statistic of 2.04. These checks show that our results are not biased by small firms.

Table (6) lists the summary statistics for the CMC factor in comparison to the market, SMB and HML factors of Fama and French (1993), the SKS coskewness factor of Harvey and Siddique (2000), and the WML momentum factor from Carhart (1997). The CMC factor has a monthly mean return of 0.23%, which is higher than the mean return of SMB (0.19% per month) and approximately two-thirds of the mean return of HML (0.32% per month). While the returns on CMC and HML are statistically significant at the 5% confidence level, the return on SMB is not statistically significant. CMC has a monthly volatility of 2.06%, which is lower than the volatilities of SMB (2.93%) and HML (2.65%). CMC also has close to zero skewness, and it is less autocorrelated (10%) than the Fama-French factors (17% for SMB and 20% for HML). The Harvey-Siddique SKS factor has a small average return per month (0.10%) and is not statistically significant. In contrast, the WML factor has the highest average return, over 0.90% per month. However, unlike the other factors, WML is constructed using equal-weighted portfolios, rather than value-weighted portfolios.

We list the correlation matrix across the various factors in Panel B of Table (6). CMC has a slightly negative correlation with the market portfolio of -16%, a magnitude less than the correlation of SMB with the market (32%) and less in absolute value than the correlation of HML with the market (-40%). CMC is positively correlated with WML (35%). The correlation matrix shows that SKS and CMC have a correlation of -3%, suggesting that asymmetric downside correlation risk has a different effect than skewness risk.

Table (6) shows that CMC is quite negatively correlated with SMB (-64%). To allay fears that CMC is not merely reflecting the inverse of the size effect, we examine the individual

firm composition of CMC and SMB. On average, 3660 firms are used to construct SMB each month, of which SMB is long 2755 firms and short 905 firms.<sup>2</sup> We find that the overlap of the firms, that SMB is going long and CMC is going short, constitutes, on average, only 27% of the total composition of SMB. Thus, the individual firm compositions of SMB and CMC are quite different. We find that the high negative correlation between the two factors stems from the fact that SMB performs poorly in the late 80's and the 90's, while CMC performs strongly over this period.

To be sure that CMC is not merely reflecting the information already captured by the market, SMB, HML and WML, Panel C of Table (6) regresses CMC on these four factors and a constant. The CMC factor loads negatively on SMB and HML, which is consistent with the correlation patterns in Panel B and the fact that high downside correlation stocks tend to be large stocks and growth stocks. The loading of CMC on WML is very small (0.07) compared to the magnitude of the SMB (-0.44) and HML (-0.21) loadings. Net of these loadings, the intercept term remains positive and significant, which indicates that CMC is not explained by the other factors. In fact, the mean return left unexplained increases to 0.33% per month, compared to the unadjusted mean return of CMC of 0.23% per month.

### 4.2 Pricing the Downside Correlation Portfolios

The 20 downside correlation portfolios examined in Section 2 cannot be explained by the market, SMB, HML and WML factors. Our CMC factor ought to explain the variations of these downside correlation portfolios, since by construction it reflects the spread in expected returns due to cross-sectional variation in  $\rho^-$ . Table (7) revisits the Fama-MacBeth (1973) regression tests in Section 2 to see if CMC is successful in pricing the downside correlation portfolios.

Model A of Table (7) is the traditional CAPM augmented with the CMC. The estimate of the premium on CMC is positive and statistically significant. Moreover, the GRS F-test cannot reject the hypothesis that the market factor augmented with CMC can price the variations in the downside correlation portfolio returns. The premium on CMC continues to remain positive and significant after adding the two Fama-French factors in Model B. It is also approximately the same as the mean CMC premium in Table (6). The GRS test suggests that some of the portfolio returns are not explained by this model, but this result is only weakly significant (p-value = 0.04). Furthermore, in the presence of the market, the Fama-French factors, and CMC factor, WML still does not affect the significance of CMC. The GRS test suggests that this model,

<sup>&</sup>lt;sup>2</sup> SMB is long more firms than it is short since the breakpoints are determined using market capitalizations of NYSE firms, even though the portfolio formation uses all NYSE, AMEX and NASDAQ firms.

which incorporates CMC, explains the variations in returns of downside correlation portfolios. In short, CMC successfully prices the downside correlation effect.

## 4.3 Forecasting Macroeconomic Variables

We briefly explore the relation between downside correlation and the business cycle by investigating how the downside correlation factor covaries with and macroeconomic variables. The investigation in this section should be regarded as an exploratory exercise, rather than as a formal test of the underlying economic determinants of downside risk. Our analysis here is motivated by studies such as Liew and Vassalou (2000), who show that other return-based factors, such as the Fama-French (1993) SMB and HML factors, can predict GDP growth and hence may reflect systematic risk.

We consider six macroeconomic variables which reflect underlying economic activity and business conditions. Our first two variables are leading indicators of economic activity: the growth rate in the index of leading economic indicators (LEI) and the growth rate in the index of Help Wanted Advertising in Newspapers (HELP). We also use the growth rate of total industrial production (IP). The next three variables measure price and term structure conditions: the CPI inflation rate, the level of the Fed funds rate (FED) and the term spread between the 10-year T-bonds and the 3-months T-bills (TERM). All growth rates (including inflation) are computed as the difference in logs of the index at times t and t-12, where t is monthly.

To examine the connection between downside risk and macroeconomic variables, we run two sets of regressions. The first set regresses CMC on lagged macro variables, while the second set regresses macroeconomic variables on lagged CMC. The first set of regressions are of the form:

$$CMC_{t} = a + \sum_{i=1}^{3} b_{i} MACRO_{t-i} + \sum_{i=1}^{3} c_{i} CMC_{t-i} + \epsilon_{t}$$
(12)

where we use various macroeconomic variables for  $MACRO_t$ .

Panel A of Table (8) lists the regression results from equation (12). There is no significant relation between lagged macroeconomic variables and the CMC factor, except for the first lag of LEI, which is significantly negatively related with CMC. A 1% increase in the growth rate of LEI predicts a 27 basis point decrease in the premium of the downside correlation factor. However, the p-value for the joint test (in the last column of Table (8)) that all lagged LEI are equal to zero fails to reject the null with p-value=0.09. Overall, with the exception of LEI, there is little evidence of predictive power by macroeconomic variables to forecast CMC returns.

To explore if the downside correlation factor predicts future movements of macroeconomic variables we run regressions of the form:

$$MACRO_t = a + \sum_{i=1}^{3} b_i CMC_{t-i} + \sum_{i=1}^{3} c_i MACRO_{t-i} + \epsilon_t.$$
 (13)

We also include lagged macroeconomic variables in the right hand side of the regression since most of the macroeconomic variables are highly autocorrelated. Panel B of Table (8) lists the regression results of equation (13). We report only the coefficients on lagged CMC. While the macroeconomic variables provide little forecasting power for CMC, the CMC factor has some forecasting ability for future macroeconomic variables. In particular, high CMC forecasts lower future economic activity (HELP, p-value = 0.00; IP, p-value = 0.03), lower future interest rates (FED, p-value = 0.01) and lower future term spreads (TERM, p-value = 0.03), where the p-values refer to a joint test that the three coefficients on lagged CMC in equation (13) are equal to zero.

In general, these results show that high CMC forecasts economic downturns. The predictions of high CMC and future low economic activity is seen directly in the negative coefficients for HELP and IP. Term spreads also tend to be lower in economic recessions. Estimates of Taylor (1993)-type policy rules on the FED over long samples, where the FED rate is a linear function of inflation and real activity, show short rates to be lower when output is low. Hence, the positive correlation of high CMC with future low HELP, low IP, low TERM and low FED shows that high CMC forecasts economic downturns. In other words, the reward for holding stocks with high downside risk is greater when future economic prospects turn sour, perhaps because the incidence of extreme market downside moves increases during recessions.

# 5 An Application to Pricing the Momentum Effect

That a CMC factor, constructed from the  $\rho^-$  portfolios, explains the cross-sectional variation across  $\rho^-$  portfolios is no surprise. Indeed, we would be concerned if the CMC factor could not price the  $\rho^-$  portfolios. As an application of the CMC factor, we demonstrate that CMC has explanatory power to account for some of the momentum effect.

#### **5.1** Data and Motivation

Why might we expect CMC to help explain some of the momentum effect? We first present evidence that momentum strategies tend to perform poorly when the market makes extreme

downward moves. To show this, we work with Jegadeesh and Titman (1993)'s momentum portfolios, corresponding to the J=6 month formation period, which is standard in the literature (Chordia and Shivakumar, 2001). After stocks are sorted into deciles based on their past 6 month returns, they are held for the next K months holding periods, where K = 3, 6, 9 or 12. We form an equal-weighted portfolio within each decile and calculate overlapping holding period returns for the next K months.

Figure (3) plots the average returns of the 40 portfolios sorted on past 6 months returns. The average returns are shown with \*'s. There are 10 portfolios corresponding to each of the K=3, 6, 9 and 12 months holding periods. Figure (3) shows average returns to be increasing across the deciles (from losers to winners) and are roughly the same for each holding period K. The differences in returns between the winner portfolio (decile 10) and the loser portfolio (decile 1) are 0.54, 0.77, 0.86 and 0.68 percent per month, with corresponding t-statistics of 1.88, 3.00, 3.87 and 3.22, for K=3, 6, 9 and 12 respectively. Hence, the return differences between winners and losers are significant at the 1% level except the momentum strategy corresponding to K=3. Figure (3) also shows the  $\rho$ <sup>-</sup> of the momentum portfolios, which increase going from the losers to the winners, except at the highest winner decile. Hence, the momentum strategies generally have a positive relation with downside correlation exposure.

Panel A of Table (9) shows the exposure of momentum strategies to downside risk. This table shows the Fama-French alphas for the losers (first decile) and winners (tenth decile) for each holding period K. The exposure of the momentum strategies to downside risk is revealed by comparing the alphas from the full sample to the alphas from a subsample where the market experiences extreme downward moves. An extreme downward move is defined to be a move that is more negative than two standard deviations below the unconditional mean. There are 41 such observations out of 432 total months.

During the full sample, the Fama-French alphas for winners are higher than losers. However, during periods of market distress, this pattern is reversed so winners perform worse than losers. The very few observations during these extreme down periods means that we should be very reluctant to conclude that winners have higher downside risk than losers, especially given the very low levels of the t-statistics. Nevertheless, the point estimates do show that losers perform better than winners during market downturns.<sup>3</sup>

We observe the same effect for the downside correlation portfolios from Table (2) in Panel B. High downside correlation stocks have higher unconditional returns than low downside

<sup>&</sup>lt;sup>3</sup> For periods when the market return is less than two standard deviations below the mean, although winners under-perform losers, the pattern moving across the decile portfolios from losers to winners is not monotonic. However, the 10th decile (winners) always underperforms the bottom decile (losers) across all *K* holding periods.

correlation stocks to compensate for their much lower returns when the market crashes. If arbitraguers are averse to downside risk in the momentum strategies, they would demand compensation into order to bear such risk. We hope that the CMC factor may pick up some of the downside exposure of the momentum portfolios. Indeed, Table (9) suggests that the momentum portfolios and the downside correlation portfolios both reflect downside risk.

Panel C of Table (9) examines the economic reduction in the momentum premium by adding a CMC factor. Model A of Table (9) regresses WML onto a constant, the MKT and the CMC factor. This regression has an  $R^2$  of 12%, and a significantly positive loading. The CMC factor reduces the momentum raw return (0.90% per month) to 0.72% per month, a reduction of 2.18% per annum. In Model B, the Fama-French (1993) WML alpha is 1.05% per month, and adding the CMC factor reduces this, in Model C, to 0.86% per month, which is a reduction of 2.30% per annum. Hence, while the momentum effect cannot be completely explained by downside correlation, Panel C of Table (9) shows that CMC has some explanatory power for WML which the other factors (MKT, SMB, HML) do not have.

#### 5.2 Fama-MacBeth (1973) Cross-Sectional Test

We now conduct formal cross-sectional estimations of the relation between downside risk and expected returns of momentum returns in Table (10) with Fama-MacBeth tests. Using data on the 40 momentum portfolios corresponding to the J=6 formation period, we first examine the Fama-French (1993) model in Model A. The estimates of the risk premia for SMB and HML are negative, which reflect the fact that the loadings on SMB and HML go the wrong way for the momentum portfolios.

In comparison, Model B adds CMC as a factor together with the market. The estimated premium on CMC is 8.76% per annum (0.73 per month) and statistically significant at the 5% level. These results do not change when SMB and HML are added in Model C. In Model D, we augment the Carhart (1997) four-factor model (MKT, SMB, HML and WML) with CMC. The factor premia on WML and CMC are both significant. The fact that CMC remains significant at the 5% level (t-stat=2.01) in the presence of WML shows that CMC is picking up some of the momentum effect reflecting downside risk. Moreover, the magnitude of the CMC factor loading remains relatively unchanged after adding WML.

Figure (4) graphs the loadings of each momentum portfolio on MKT, SMB, HML and CMC. The loadings are estimated from the time-series regressions of the momentum portfolios on the factors from the first step of the Fama-MacBeth (1973) procedure. We see that for each set of portfolios, as we go from the past loser portfolio (decile 1) to the past winner portfolio

(decile 10), the loadings on the market portfolio remain flat, so that beta has little explanatory power. The loadings on SMB decrease from the losers to the winners, except for the last two deciles. Similarly, the loadings on the HML factor also go in the wrong direction, decreasing monotonically from the losers to the winners.

In contrast to the decreasing loadings on the SMB and HML factors, the loadings on the CMC factor in Figure (4) almost monotonically increase from strongly negative for the past loser portfolios to slightly positive for the past winner portfolios. The increasing loadings on CMC across the decile portfolios for each holding period K are consistent with the increasing  $\rho^-$  statistics across the deciles in Figure (3). Winner portfolios have higher  $\rho^-$ , higher loadings on CMC, and higher expected returns. The negative loadings for loser stocks imply that losers have higher downside correlation exposure than winners. This reflects the evidence in Table (9), which shows that past winner stocks do poorly when the market has large moves on the downside, while past loser stocks perform better in these extreme periods.

#### 5.3 GMM Hypothesis Tests

Using GMM cross-sectional estimations (described in the Appendix), we can conduct some additional hypothesis tests for the goodness of fit for the various models in Table (10) to price the momentum effect. Taking Model C as an unconstrained model and using its weighting matrix to re-estimate Model A, we can conduct a  $\chi^2$  over-identification test. This tests rejects the null hypothesis of the Fama-French model with a p-value=0.02. Hence, CMC does provide additional explanatory power for the cross-section of momentum portfolios which the Fama-French model does not provide. Model D of Table (10) nests Model C, which uses MKT, SMB, HML and CMC factors. We run a  $\chi^2$  over-identification test with the null of Model C against the alternative of Model D, which rejects with a p-value of 0.01. Hence, we conclude that WML still has further explanatory power, in the presence of CMC, to price the cross-section of momentum portfolios.

We graph the average pricing errors for the models in Figure (5), following Hodrick and Zhang (2001). The pricing errors are computed using the weighting matrix,  $W_T = \mathrm{E}[R_t R_t']^{-1}$ , where  $R_t$  is a vector of gross returns of the base assets. Since the same weighting matrix is used across all of the models, we can compare the differences in the pricing errors for different models. Figure (5) displays each momentum portfolio on the x-axis, where the first ten portfolios correspond to the K=3 month holding period, the second ten to the K=6 month holding period, the third ten to the K=9 month holding period, and finally the fourth ten to the K=12 holding period. The 41st asset is the risk-free asset. The figure plots two

standard error bounds in solid lines, and the pricing errors for each asset in \*'s.

Figure (5) shows that the CAPM has most of its pricing errors outside the two standard error bands and shows that the loser portfolios are the most difficult for the CAPM to price. The Fama-French model has most difficulty pricing past winners; the pricing errors of every highest winner portfolio lies outside the two standard error bands. The model using MKT and CMC factors is the only model that has all the pricing errors within two standard error bands. However, adding CMC to the Fama French model or the Carhart model does not change the pricing errors of the assets very much.

While Figure (5) can give us a visual representation of the pricing errors, we can formally test if all the pricing errors are zero by using a Hansen-Jagannathan (1997) (HJ) test (see the Appendix for further details). This tests overwhelmingly rejects, both asymptotically and with small-sample simulations, the null that the pricing errors are jointly zero for all the models. Although all pricing errors for the model of MKT and CMC fall within the two standard error bands, the HJ tests reject this model because because the HJ distance does not assign an equal weight to all the portfolios in the test. Hence, while momentum portfolios do seem to have exposure to downside risk, the CMC factor only modestly reduces the momentum premium by 2.18%, and although it comes out significant in cross-sectional tests with momentum assets, CMC cannot completely account for all the momentum effect.

## 6 Conclusion

We find that stocks with high downside correlations have higher expected returns than stocks with low downside correlations. The portfolio of stocks with the greatest downside correlations outperforms the portfolio of stocks with the lowest downside correlations by 4.91% per annum. Downside correlation is distinct from market risk and liquidity risk, and it is not mechanically linked to past returns. Moreover, controlling for the market beta, the size effect and the book-to-market effect increases the difference in the returns between the highest and the lowest downside correlation portfolios to 6.55% per annum. To capture this asymmetry, we construct a downside correlation factor (CMC) that goes long stocks with high downside correlations and goes short stocks with low downside correlations. The CMC factor is priced by portfolios of stocks sorted by downside correlations, and exposure to this downside correlation factor helps explain some of the profitability of momentum strategies.

While most economic models would suggest that both the magnitude and the direction of risk ought to matter, our downside correlation measure only captures the direction of downside comovements, but not the magnitude of the comovements. The decomposition of the betas show that the action of total and market volatilities confound the magnitudes of joint downside movements. In particular, the ratio of total to market volatility decreases on the downside, which causes conditional betas to exhibit little asymmetry across the downside and the upside. In contrast, conditional correlations are immune to these effects and exhibit highly asymmetric patterns across downside and upside market movements. Hence, conditional correlations appear to be a cleaner measure of asymmetric exposure to risk than conditional betas.

While we show that high downside correlation stocks command high expected returns that cannot be accounted for by the standard risk factors, our empirical work leaves unexplained what underlying economic mechanisms cause some stocks to exhibit greater downside risk, and why investors demand compensation for exposures to such risk. A more difficult task is capturing the interaction between idiosyncratic and market volatility across up and down markets which causes conditional betas to have little asymmetry across downside and upside markets, while conditional correlations exhibit pronounced asymmetry across downside and upside markets. Economies with frictions and hidden information (Hong and Stein, 2001) or with agents facing binding wealth constraints (Kyle and Xiong, 2001) have both significant direction and magnitude effects. Although representative agent models with asymmetric utility functions, like first-order risk aversion (Bekaert, Hodrick and Marshall, 1997) or loss aversion (Barberis, Huang and Santos, 2001), have not been calibrated in the cross-section, these models would also assign a larger role to conditional betas than to conditional correlations.

# **Appendix**

## A Data and Portfolio Construction

#### **Data Sources**

We use data from the Center for Research in Security Prices (CRSP) to construct portfolios of stocks sorted by various characteristics of returns. We confine our attention to ordinary common stocks listsed on NYSE, AMEX and NASDAQ, omitting ADRs, REITs, closed-end funds, foreign firms and other securities which do not have a CRSP share type code of 10 or 11. We use daily returns from CRSP for the period covering January 1st, 1964 to December 31st, 1999, including NASDAQ data which is only available post-1972. We use the one-month risk-free rate from CRSP and take CRSP's value-weighted returns of all stocks as the market portfolio. All our returns are expressed as continuously compounded returns.

The 48 industry portfolios are from Fama and French (1997) and are obtained from Kenneth French's website at http://web.mit.edu/kfrench/www/data\_library.html. The Fama and French (1993) factors, SMB and HML, are also from the data library at Kenneth French's website.

#### **Higher Moment Portfolios**

We construct portfolios based on correlations between asset i's excess return  $r_{it}$  and the market's excess return  $r_{mt}$  conditional on downside moves of the market ( $\rho^-$ ) and on upside moves of the market ( $\rho^+$ ). We also constuct portfolios based on coskewness, cokurtosis,  $\beta$ ,  $\beta$  conditional on downside market movements ( $\beta^-$ ), and  $\beta$  conditional on upside market movements ( $\beta^+$ ).

Coskewness is defined, following Harvey and Siddique (2000), as:

$$\operatorname{coskew} = \frac{\mathrm{E}[\epsilon_{i,t}\epsilon_{m,t}^2]}{\sqrt{\mathrm{E}[\epsilon_{i,t}^2]\mathrm{E}[\epsilon_{m,t}^2]}},\tag{A-1}$$

where  $\epsilon_{i,t} = r_{i,t} - \alpha_i - \beta_i M K T_t$ , is the residual from the regression of  $r_{i,t}$  on the contemporaneous excess market return, and  $\epsilon_{m,t}$  is the residual from the regression of the market excess return on a constant. Similar to the definition of coskewness in equation (A-1), we define cokurtosis as:

$$\operatorname{cokurt} = \frac{\operatorname{E}[\epsilon_{i,t}\epsilon_{m,t}^3]}{\sqrt{\operatorname{E}[\epsilon_{i,t}^2]} \left(\operatorname{E}[\epsilon_{m,t}^2]\right)^{\frac{3}{2}}}.$$
(A-2)

At the beginning of each month, we calculate each stock's moment measures using the past year's daily log returns from the CRSP daily file. For the moments which condition on downside or upside movements, we define an observation at time t to be a downside (upside) market movement if the excess market return at t is less than or equal to (greater than or equal to) the average excess market return during the past one year period in consideration. We require a stock to have at least 220 observations to be included in the calculation. These moment measures are then used to sort the stocks into deciles and a value-weighted return is calculated for all stocks in each decile. The portfolios are rebalanced monthly.

#### **SKS and WML Factor Construction**

Harvey and Siddique (2000) use 60 months of data to compute the coskewness defined in equation (A-1) for all stocks in NYSE, AMEX and NASDAQ. Stocks are sorted in order of increasing negative coskewness. The coskewness factor SKS is the value-weighted average returns of firms in the top 3 deciles (with the most negative coskewness) minus the value-weighted average return of firms in the bottom 3 deciles (stocks with the most positive coskewness) in the 61st month.

Following Carhart (1997), we construct WML as the equally-weighted average of firms with the highest 30 percent eleven-month returns lagged one month minus the equally-weighted average of firms with the lowest 30 percent eleven-month returns lagged one month. In constructing WML, all stocks in NYSE, AMEX and NASDAQ are used and portfolios are rebalanced monthly.

### **Liquidity Factor and Liquidity Betas**

We follow Pástor and Stambaugh (2001) to construct an aggregate liquidity measure, L. Stock return and volume data are obtained from CRSP. NASDAQ stocks are excluded in the construction of the aggregate liquidity measure. The liquidity estimate,  $\gamma_{i,t}$ , for an individual stock i in month t is the ordinary least squares (OLS) estimate of  $\gamma_{i,t}$  in the following regression:

$$r_{i,d+1,t}^{e} = \theta_{i,t} + \phi_{i,t}\bar{r}_{i,d,t} + \gamma_{i,t}sign\left(r_{i,d,t}^{e}\right)v_{i,d,t} + \epsilon_{i,d+1,t}, \quad d = 1,\dots, D. \tag{A-3}$$

In equation (A-3),  $\bar{r}_{i,d,t}$  is the raw return on stock i on day d of month t,  $r_{i,d,t}^e = r_{i,d,t} - r_{m,d,t}$  is the stock return in excess of the market return, and  $v_{i,d,t}$  is the dollar volume for stock i on day d of month t. The market return on day on day d of month t,  $r_{m,d,t}$ , is taken as the return on the CRSP value-weighted market portfolio. A stock's liquidity estimate,  $\gamma_{i,t}$ , is computed in a given month only if there are at least 15 consecutive observations, and if the stock has a month-end share prices of greater than \$5 and less than \$1000.

The aggregate liquidity measure, L, is computed based on the liquidity estimates,  $\gamma_{i,t}$ , of individual firms listed on NYSE and AMEX from August 1962 to December 1992. Only the individual liquidity estimates that meet the above criteria is used. To construct the innovations in aggregate liquidity, we follow Pástor and Stambaugh and first form the scaled monthly difference:

$$\Delta \hat{\gamma}_t = \left(\frac{m_t}{m_1}\right) \frac{1}{N} \sum_{i=1}^N (\gamma_{i,t} - \gamma_{i,t-1}),\tag{A-4}$$

where N is the number of available stocks at month t,  $m_t$  is the total dollar value of the included stocks at the end of month t-1, and  $m_1$  is the total dollar value of the stocks at the end of July 1962. The innovations in liquidity are computed as the residuals in the following regression:

$$\Delta \hat{\gamma}_t = a + b \Delta \hat{\gamma}_{t-1} + c (m_t/m_1) \hat{\gamma}_{t-1} + u_t. \tag{A-5}$$

Finally, the aggregate liquidity measure,  $L_t$ , is taken to be the fitted residuals,  $L_t = \hat{u}_t$ .

To calculate the liquidity betas for individual stocks, at the end of each month between 1968 and 1999, we identify stocks listed on NYSE, AMEX and NASDAQ with at least five years of monthly returns. For each stock, we estimate a liquidity beta,  $\beta_i^L$ , by running the following regression using the most recent five years of monthly data:

$$r_{i,t} = \beta_i^0 + \beta_i^L L_t + \beta_i^M M K T_t + \beta_i^S S M B_t + \beta_i^H H M L_t + \epsilon_{i,t}, \tag{A-6}$$

where  $r_{i,t}$  denotes asset i's excess return and  $L_t$  is the innovation in aggregate liquidity.

#### **Momentum Portfolios**

To construct the momentum portfolios of Jegadeesh and Titman (1993), we sort stocks into portfolios based on their returns over the past 6 months. We consider holding period of 3, 6, 9 and 12 months. This procedure yields 4 strategies and 40 portfolios in total. We illustrate the construction of the portfolios with the example of the '6-6' strategies. To construct the '6-6' deciles, we sort our stocks based upon the past six-months returns of all stocks in NYSE and AMEX. Each month, an equal-weighted portfolio is formed based on six-months returns ending one month prior. Similarly, equal-weighted portfolios are formed based on past returns that ended one months prior, three months prior, and so on up to six months prior. We then take the simple average of six such portfolios. Hence, our first momentum portfolio consists of 1/6 of the returns of the worst performers one month ago, plus 1/6 of the returns of the worst performers two months ago, etc.

#### **Macroeconomic Variables**

We use the following macroeconomic variables from the Federal Reserve Bank of St. Louis: the growth rate in the index of leading economic indicators (LEI), the growth rate in the index of Help Wanted Advertising in Newspapers (HELP), the growth rate of total industrial production (IP), the Consumer Price Index inflation rate (CPI), the level of the Fed funds rate (FED), and the term spread between the 10-year T-bonds and the 3-months T-bills (TERM). All growth rates (including inflation) are computed as the difference in logs of the index at times t and t-12, where t is monthly.

## **B** Fama-MacBeth and GMM Cross-Sectional Tests

#### Fama-MacBeth (1973) Cross-Sectional Tests

We consider linear cross-sectional regressional models of the form:

$$E(r_{it}) = \lambda_0 + \lambda' \beta_i, \tag{B-1}$$

in which  $\lambda_0$  is a scalar,  $\lambda$  is a  $M \times 1$  vector of factor premia, and  $\beta_i$  is an  $M \times 1$  vector of factor loadings for portfolio i. The Fama-MacBeth (1973) is a two-step cross-sectional estimation procedure.

In the first step, we use the entire sample to estimate the factor loadings,  $\beta_i$ :

$$r_{it} = \alpha_i + F_t' \beta_i + \varepsilon_{it}, \quad t = 1, 2, \dots T, \tag{B-2}$$

where  $\alpha_i$  is a scalar and  $F_t$  is a  $M \times 1$  vector of factors. In the second step, we run a cross-sectional regression at each time t over N portfolios, holding the  $\beta_i$ 's fixed at their estimated values,  $\hat{\beta}_i$ , in equation (B-2):

$$r_{it} = \lambda_0 + \lambda' \hat{\beta}_i + u_{it}, \quad i = 1, 2, ...N.$$
 (B-3)

The factor premia,  $\lambda$ , are estimated as the averages of the cross-sectional regression estimates:

$$\hat{\lambda} = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_t. \tag{B-4}$$

The covariance matrix of  $\lambda$ ,  $\Sigma_{\lambda}$ , is estimated by:

$$\hat{\Sigma}_{\lambda} = \frac{1}{T^2} \sum_{t=1}^{T} (\hat{\lambda}_t - \bar{\lambda})(\hat{\lambda}_t - \bar{\lambda})', \tag{B-5}$$

where  $\bar{\lambda}$  is the mean of  $\lambda$ .

Since the factor loadings are estimated in the first stage and these loadings are used as independent variables in the second stage, there is an errors-in-variables problem. To remedy this, we use Shanken's (1992) method to adjust the standard errors by multiplying  $\hat{\Sigma}_{\lambda}$  with the adjustment factor  $(1+\hat{\lambda}'\hat{\Sigma}_f^{-1}\hat{\lambda})^{-1}$ , where  $\hat{\Sigma}_f$  is the estimated covariance matrix of the factors  $F_t$ .

#### **GMM Cross-Sectional Tests**

The standard Euler equation for a gross return,  $R_{it}$ , is given by:

$$E(m_t R_{it}) = 1. (B-6)$$

Linear factor models assume that the pricing kernel can be written as a linear combination of factors:

$$m_t = \delta_0 + \delta_1' F_t, \tag{B-7}$$

where  $F_t$  is a  $M \times 1$  vector of factors,  $\delta_0$  is a scalar, and  $\delta_1$  is a  $M \times 1$  vector of coefficients. The representation in equation (B-7) is equivalent to a linear beta pricing model:

$$E(R_{it}) = \lambda_0 + \lambda' \beta_i, \tag{B-8}$$

which is analogous to equation (B-1) for excess returns. The constant  $\lambda_0$  is given by:

$$\lambda_0 = \frac{1}{\mathrm{E}(m_t)} = \frac{1}{\delta_0 + \delta_1' E(F_t)},$$

the factor loadings,  $\beta_i$ , are given by:

$$\beta_i = \operatorname{cov}(F_t, F_t')^{-1} \operatorname{cov}(F_t, R_{it}),$$

and the factor premia,  $\lambda$ , are given by:

$$\lambda = -\frac{1}{\delta_0} \text{cov}(F_t, F_t') \delta_1.$$

To test whether a factor j is priced, we test the null hypothesis  $H_0: \lambda_i = 0$ .

Letting  $R_t$  denote an  $N \times 1$  vector of gross returns  $R_t = (R_{1t}, \dots, R_{Nt})'$ , and denoting the parameters of the pricing kernel as  $\delta = (\delta_0, \delta_1')'$ , the sample pricing error is:

$$g_T(\delta) = \frac{1}{T} \sum_{t=1}^{T} (m_t R_t - 1).$$
 (B-9)

The GMM estimate of  $\delta$  is the solution to:

$$\min_{\delta} J = T \times g_T' W_T g_T, \tag{B-10}$$

where  $W_T$  is a weighting matrix.

#### Hansen-Jagannathan (1997) Test

Jagannathan and Wang (1996) derive the asymptotic distribution of the HJ distance metric:

$$HJ = \sqrt{g_T(\delta)' \mathbb{E}[R_t R_t']^{-1} g_T(\delta)}, \tag{B-11}$$

which can be interpreted as the least-square distance between a given pricing kernel and the closest point in the set of the pricing kernels that can price the base assets correctly. The asymptotic distribution of  $T \times (HJ)^2$  involves a weighted sum of (N-K-1)  $\chi_1^2$  statistics. The weights are the N-K-1 non-zero eigenvalues of:

$$A = S_T^{\frac{1}{2}} W_T^{\frac{1}{2}'} \left[ I - W_T^{\frac{1}{2}} D_T (D_T' W_T D_T)^{-1} D_T' W_T^{\frac{1}{2}'} \right]^{-1} W_T^{\frac{1}{2}} S_T^{\frac{1}{2}'},$$

where  $S_T^{\frac{1}{2}}$  and  $W_T^{\frac{1}{2}}$  are the upper-triangular Cholesky decompositions of  $S_T$  and  $W_T$  respectively, and  $D_T = \frac{\partial g_T}{\partial \delta}$ . The matrix  $S_T$  is the optimal weighting matrix, where  $W_T^* = S_T^{-1} = [T \cdot \text{cov}(g_T, g_T')]^{-1}$ . Jagannathan and Wang show that A has exactly N-K-1 positive eigenvalues  $\theta_1, \ldots, \theta_{N-K-1}$ . The asymptotic distribution of the HJ distance metric is:

$$T \times (HJ)^2 \to \sum_{j=1}^{N-K-1} \theta_j \chi_1^2$$

as  $T \to \infty$ . We simulate the HJ statistic 100,000 times to compute the asymtotic p-value of the HJ distance.

To calculate a small sample p-value for the HJ distance, we assume that the linear factor model holds and simulate a data generating process (DGP) with 432 observations, the same length as in our samples. The DGP takes the form:

$$r_{i,t} = r_{t-1}^f + \beta_i' F_t + \epsilon_{it},$$
 (B-12)

where  $r_{i,t}$  is the return on the i-th portfolio,  $r_t^f$  is the risk-free rate,  $\beta_i$  is an  $M \times 1$  vector of factor loadings, and  $F_t$  is the  $M \times 1$  vector of factors. We assume that the risk-free rate and the factors follow a first-order VAR process. Let  $X_t = (r_t^f, F_t)'$ , and  $X_t$  follows:

$$X_t = \mu + AX_{t-1} + u_t, (B-13)$$

where  $u_t \sim N(0, \Sigma)$ . We estimate this VAR system and use the estimates  $\hat{\mu}$ ,  $\hat{A}$  and  $\hat{\Sigma}$  as the parameters for our factor generating process. In each simulation, we generate 432 observations of factors and the risk-free rate from the VAR system in equation (B-13). For the portfolio returns, we use the sample regression coefficient of each portfolio return on the factors,  $\hat{\beta}_i$ , as our factor loadings. We assume the error terms of the base assets,  $\epsilon_t$ , follow IID multivariate normal distributions with mean zero and covariance matrix,  $\hat{\Sigma}_r - \hat{\beta}' \hat{\Sigma}_F \hat{\beta}$ , where  $\hat{\Sigma}_r$  is the covariance matrix of the assets and  $\hat{\Sigma}_F$  is the covariance matrix of the factors.

For each model, we simulate 5000 time-series as described above and compute the HJ distance for each simulation run. We then count the percentage of these HJ distances that are larger than the actual HJ distance from real data and denote this ratio empirical p-value. For each simulation run, we also compute the theoretic p-value which is calculated from the asymptotic distribution.

## References

- [1] Ang, A., and J. Chen, 2001, "Asymmetric Correlations of Equity Returns," forthcoming *Journal of Financial Economics*.
- [2] Bansal, R., D. A. Hsieh, and S. Viswanathan, 1993, "A New Approach to International Arbitrage Pricing," Journal of Finance, 48, 1719-1747.
- [3] Barberis, N., A. Shleifer, and R. Vishny, 1998, "A Model of Investor Sentiment," *Journal of Financial Economics*, 49, 3, 307-343.
- [4] Barberis, N., Huang, M., and T. Santos, 2001, "Prospect Theory and Asset Prices," *Quarterly Journal of Economics*, 116, 1, 1-53.
- [5] Bawa, V. S., and E. B. Lindenberg, 1977, "Capital Market Equilibrium in a Mean-Lower Partial Moment Framework," *Journal of Financial Economics*, 5, 189-200.
- [6] Bekaert, G., R. J. Hodrick, and D. A. Marshall, 1997, "The Implications of First-Order Risk Aversion for Asset Market Risk Premiums," *Journal of Monetary Economics*, 40, 3-39.
- [7] Campbell, J., M. Lettau, B. G. Malkiel and Y. Xu, 2001, "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *Journal of Finance*, 56, 1-44.
- [8] Carhart, M. M. 1997, "On Persistence in Mutual Fund Performance," Journal of Finance, 52, 1, 57-82.
- [9] Chen, J., Hong, H., and J. Stein, 2001, "Breadth of Ownership and Stock Returns," forthcoming *Journal of Financial Economics*.
- [10] Chen, N., R. Roll, and S. Ross, 1986, "Economic Forces and the Stock Market," *Journal of Business*, 59, 383-403.
- [11] Chordia, T., R. Roll, and A. Subrahmanyam, 2000, "Market Liquidity and Trading Activity," forthcoming *Journal of Finance*.
- [12] Chordia, T., and L. Shivakumar, 2001, "Momentum, Business Cycle and Time-Varying Expected Returns," forthcoming *Journal of Finance*.
- [13] Cochrane, J. H., 1996, "A Cross-Sectional Test of an Investment Based Asset Pricing Model," *Journal of Political Economy*, 104, 572-621.
- [14] Daniel, K., D. Hirshleifer, and A. Subrahmanyam, 1998, "Investor Psychology and Security Market Underand Overreactions," *Journal of Finance*, 53, 1839-1886.
- [15] DeBondt, Werner F. M. and Thaler, R. H., 1987, "Further Evidence on Investor Overreaction and Stock Market Seasonality," *Journal of Finance*, 42, 557-581.
- [16] Dittmar, R., 2001, "Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross-Section of Equity Returns," forthcoming *Journal of Finance*.
- [17] Duffee, G., 1995, "Asymmetric Cross-Sectional Dispersion in Stock Returns: Evidence and Implications," working paper.
- [18] Fama, E. F., and J. D. MacBeth, 1973, "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy*, 71, 607-636.
- [19] Fama, E. F., and K. R. French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 33, 3-56.
- [20] Fama, E. F., and K. R. French, 1996, "Multifactor Explanations of Asset Pricing Anomalies," Journal of Finance, 51, 1, 55-84.
- [21] Fama, E. F., and K. R. French, 1997, "Industry Costs of Equity," Journal of Financial Economics, 43, 153-193.
- [22] Gervais, S., R. Kaniel, and D. Mingelgrin, 2001, "The High Volume Return Premium," forthcoming *Journal of Finance*.
- [23] Gibbons, M. R., S. A. Ross, and J. Shanken, 1989, "A Test of the Efficiency of a Given Portfolio," *Econometrica*, 57, 5, 1121-1152.

- [24] Grundy, B. D., and S. J. Martin, 2001, "Understanding the Nature of Risks and the Sources of Rewards to Momentum Investing," *Review of Financial Studies*, 14, 1, 29-78.
- [25] Gul, F., 1991, "A Theory of Disappointment Aversion," Econometrica, 59, 3, 667-686.
- [26] Hansen, L. P., 1982, "Large Sample Properties of Generalized Method of Moments Estimators," Econometrica, 50, 1029-1054
- [27] Hansen, L. P., and R. Jagannathan, 1997, "Assessing Specification Errors in Stochastic Discount Factor Models," *Journal of Finance*, 52, 2, 557-590.
- [28] Harvey, C. R., and A. Siddique, 2000, "Conditional Skewness in Asset Pricing Tests," *Journal of Finance*, 55, 3, 1263-1295.
- [29] Hirshleifer, D., 2001, "Investor Psychology and Asset Pricing," Journal of Finance, 56, 4, 1533-1597.
- [30] Hodrick, R. J., and X. Zhang, 2001, "Evaluating the Specification Errors of Asset Pricing Models," forthcoming *Journal of Financial Economics*.
- [31] Hong, H., and J. Stein, 1999, "A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets," *Journal of Finance*, 54, 6, 2143-2185.
- [32] Hong, H., and J. Stein, 2001, "Differences of Opinion, Rational Arbitrage and Market Crashes," working paper, Stanford University.
- [33] Jagannathan, R., and Z. Wang, 1996, "The Conditional CAPM and the Cross-Section of Expected Returns," Journal of Finance, 51, 3-53.
- [34] Jegadeesh, N., and S. Titman, 1993, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance*, 48, 65-91.
- [35] Jones, C., 2001, "A Century of Stock Market Liquidity and Trading Costs," working paper Columbia University.
- [36] Kahneman, D., and A. Tversky, 1979, "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica*, 47, 263-291.
- [37] Kraus, A., and R. H. Litzenberger, 1976, "Skewness Preference and the Valuation of Risk Assets," *Journal of Finance*, 31, 4, 1085-1100.
- [38] Kyle, A. W., and W. Xiong, 2001, "Contagion as a Wealth Effect of Financial Intermediaries," *Journal of Finance*, 56, 1401-1440.
- [39] Lamont, O., C. Polk, and J. Saá-Requejo, 2001, "Financial Constraints and Stock Returns," *Review of Financial Studies*, 14, 2, 529-554.
- [40] Liew, J., and M. Vassalou, 2000, "Can Book-to-Market, Size and Momentum be Risk Factors that Predict Economic Growth?" *Journal of Financial Economics*, 57, 221-245.
- [41] Markowitz, H., 1959, Portfolio Selection. New Haven, Yale University Press.
- [42] Newey, W. K., and K. D. West, 1987, "A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703-8.
- [43] Pástor, L., and R. F. Stambaugh, 2001, "Liquidity Risk and Expected Stock Returns," working paper, Wharton.
- [44] Shanken, J., 1992, "On the Estimation of Beta-Pricing Models," Review of Financial Studies, 5, 1-33.
- [45] Taylor, J. B., 1993, "Discretion versus policy rules in practice." Carnegie-Rochester Conference Series on Public Policy, 39, 195-214.

Table 1: Portfolios Sorted on Past  $\beta$ ,  $\beta^-$  and  $\beta^+$ 

Panel A: Portfolios Sorted on Past  $\beta$ 

Portfolio 1 Low $\beta$ 2 3 4 5 6 7 8 9 10 High $\beta$	Mean 0.90 0.93 1.01 0.95 1.13 1.02 1.00 0.97 1.07 1.13	Std 3.72 3.19 3.33 3.62 3.78 3.84 4.37 4.87 5.80 7.63	Auto 0.13 0.20 0.18 0.14 0.08 0.06 0.07 0.07 0.08 0.05	$\beta$ 0.42 0.49 0.59 0.70 0.76 0.79 1.04 1.23 1.57				High–Low 0.23	t-stat 0.70
		Panel	B: Por	tfolios	Sorted	on Pas	$t \beta^-$		
Portfolio	Mean	Std	Auto	$\beta$	$\beta^-$	$ ho^-$	$k^-$	High-Low	t-stat
1 Low $\beta^-$	0.78	4.21	0.16	0.67	0.89	0.71	1.26	0.31	1.04
2	0.93	3.74	0.14	0.68	0.74	0.73	1.02		
3	0.99	3.71	0.09	0.73	0.82	0.83	0.98		
4	1.09	3.92	0.05	0.80	0.88	0.89	0.99		
5	1.05	4.00	0.06	0.85	0.89	0.91	0.98		
6	1.06	4.52	0.07	0.98	0.98	0.93	1.06		
7	1.11	4.82	0.04	1.04	1.02	0.92	1.11		
8	1.24	5.39	0.05	1.17	1.12	0.92	1.21		
9	1.22	6.26	0.04	1.32	1.30	0.89	1.46		
10 High $\beta^-$	1.09	7.81	0.08	1.57	1.52	0.84	1.82		
		Panel	C: Por	tfolios	Sorted	on Pas	t β+		
Portfolio	Mean	Std	Auto	$\beta$	$\beta^+$	$ ho^+$	$k^+$	High-Low	t-stat
1 Low $\beta^+$	1.05	5.46	0.16	0.93	0.77	0.46	1.67	-0.05	-0.21
	1.06	4.33	0.19	0.83	0.67	0.59	1.14	0.02	0.21
2 3	1.05	4.06	0.16	0.80	0.69	0.67	1.04		
4	1.01	4.10	0.11	0.83	0.82	0.75	1.09		
5	0.98	4.03	0.13	0.84	0.79	0.75	1.05		
6	1.05	4.07	0.06	0.87	0.86	0.84	1.02		

The table lists summary statistics for value-weighted  $\beta$ ,  $\beta^-$  and  $\beta^+$  portfolios at a monthly frequency, where  $\beta^-$  and  $\beta^+$  are defined in equation (3), setting  $\theta = \overline{MKT}$ . For each month, we calculate  $\beta$  ( $\beta^-$ ,  $\beta^+$ ) of all stocks based on daily continuously compounded returns over the past year. We rank the stocks into deciles (1–10), and calculate the value-weighted simple percentage return over the next month. We rebalance the portfolios monthly. Means and standard deviations are in percentage terms per month. Std denotes the standard deviation (volatility), Auto denotes the first autocorrelation, and  $\beta$  is post-formation the beta of the portfolio. The columns labeled  $\beta^-$  ( $\beta^+$ ) and  $\rho^-$  ( $\rho^+$ ) show the post-formation downside (upside) betas and downside (upside) correlations of the portfolios. The column labeled  $k^+$  ( $k^-$ ) lists the ratio of the volatility of the portfolio to the volatility of the market, both conditioning on the downside (upside). High–Low is the mean return difference between portfolio 10 and portfolio 1 and t-stat gives the t-statistic for this difference. T-statistics are computed using Newey-West (1987) heteroskedastic-robust standard errors with 3 lags. The sample period is from January 1964 to December 1999.

7

8

10 High  $\beta^+$ 

1.07

1.02

1.12

1.00

4.35

4.65

5.25

6.77

0.06

0.04

0.05

0.06

0.94

1.01

1.12

1.41

0.90

0.98

1.13

1.45

0.86

0.88

0.86

0.80

1.05

1.11

1.31

1.81

Table 2: Portfolios Sorted on Conditional Correlations

Panel A: Portfolios Sorted on Past  $\rho^-$ 

Portfolio	Mean	Std	Auto	$\beta$	FF $\alpha$	Size	B/M	Lev	$\rho^-$	$\beta^-$
1 Low $\rho^-$	0.77	4.18	0.15	0.69	-0.37	2.61	0.63	4.96	0.74	0.94
2	0.88	4.34	0.17	0.81	-0.30	2.92	0.62	4.71	0.80	0.97
3	0.87	4.32	0.15	0.83	-0.31	3.19	0.60	4.88	0.82	0.95
4	0.94	4.39	0.15	0.87	-0.24	3.46	0.58	4.38	0.83	0.97
5	0.97	4.39	0.10	0.90	-0.19	3.74	0.56	5.15	0.85	0.95
6	1.00	4.45	0.09	0.94	-0.16	4.04	0.53	4.49	0.90	1.01
7	1.00	4.64	0.09	1.00	-0.15	4.39	0.50	7.06	0.92	1.02
8	1.03	4.58	0.08	1.00	-0.10	4.82	0.48	4.04	0.94	1.05
9	1.12	4.77	0.02	1.05	0.02	5.36	0.46	5.42	0.96	1.08
10 High $ ho^-$	1.17	4.76	0.01	1.04	0.16	6.38	0.39	4.40	0.94	0.97
High - Low	0.40	2.26								
mgn - Low	0.40	4.40								

Panel B: Portfolios Sorted on Past  $\rho^+$ 

Portfolio	Mean	Std	Auto	$\beta$	FF $\alpha$	Size	B/M	Lev	$ ho^-$	$\beta^{-}$
1 Low $\rho^+$	1.13	4.56	0.17	0.82	0.02	2.88	0.60	7.52	0.50	0.63
2	1.05	4.63	0.19	0.90	-0.13	3.08	0.59	6.18	0.63	0.78
3	1.09	4.61	0.16	0.92	-0.10	3.24	0.58	4.51	0.68	0.82
4	1.06	4.67	0.15	0.94	-0.13	3.44	0.56	4.57	0.70	0.85
5	0.99	4.62	0.14	0.95	-0.18	3.66	0.54	4.47	0.76	0.91
6	1.03	4.62	0.12	0.97	-0.17	3.91	0.54	4.42	0.78	0.90
7	1.00	4.70	0.09	1.01	-0.18	4.23	0.52	4.84	0.84	0.97
8	1.11	4.67	0.08	1.01	-0.06	4.65	0.52	4.25	0.85	0.96
9	1.12	4.63	0.07	1.02	-0.01	5.27	0.48	4.71	0.92	1.02
10 High $ ho^+$	1.07	4.52	0.00	1.00	0.07	6.65	0.36	4.35	0.95	1.06

High - Low -0.06 -0.38

The table lists summary statistics of the value-weighted  $\rho^-$  and  $\rho^+$  portfolios at a monthly frequency, where  $\rho^-$  and  $\rho^+$  are defined in equation (5), setting  $\theta = \overline{MKT}$ . For each month, we calculate  $\rho^-$  ( $\rho^+$ ) of all stocks based on daily continuously compounded returns over the past year. We rank the stocks into deciles (1–10), and calculate the value-weighted simple percentage return over the next month. We rebalance the portfolios at a monthly frequency. Means and standard deviations are in percentage terms per month. Std denotes the standard deviation (volatility), Auto denotes the first autocorrelation, and  $\beta$  is the post-formation beta of the portfolio with respect to the market portfolio. The column labeled FF  $\alpha$  lists the  $\alpha$  from a regression of the excess  $\rho^-$  portfolio return on the Fama-French (1993) model at a monthly frequency. At the beginning of each month t, we compute each portfolio's simple average log market capitalization in millions (size) and value-weighted book-to-market ratio (B/M). The column Lev is the simple average of firms leverage ratio which is defined as the ratio of book value of asset to book value of equity. The columns labeled  $\rho^-$  ( $\rho^+$ ) and  $\beta^-$  ( $\beta^+$ ) show the post-formation downside (upside) correlations and downside (upside) betas of the portfolios. High-Low is the mean return difference between portfolio 10 and portfolio 1 and t-stat gives the t-statistic for this difference. T-statistics are computed using Newey-West (1987) heteroskedastic-robust standard errors with 3 lags. The sample period is from January 1964 to December 1999.

Table 3: Pricing the Downside Correlation Portfolios (1)

#### Factor Premiums $\lambda$

	$\lambda_0$	MKT	SMB	HML	WML	GRS Test
Model A: Fan	na-Fren	ch Mode	el			
Premium $(\lambda)$ t-stat	0.54 2.41*	0.65 0.20	-0.46 -1.78	0.06 0.19		3.24 p-val=0.00**
Model B: Fan	na-Fren	ch Facto	rs and V	WML		
Premium $(\lambda)$ t-stat	0.49 2.03*	0.12 0.36	-0.43 -1.60	0.13 0.39	0.69 0.99	2.31 p-val=0.00 **

The table shows the results from Fama-MacBeth (1973) regression tests on 20 downside correlation portfolios. These portfolios are formed in the following fashion. Stocks are first sorted into two groups according to their past beta over the past year, using daily returns (high beta versus low beta). Each group consists of one half of all firms. Then, within each beta group, we rank stocks based on their  $\rho^-$ , also computed using daily data over the past year into decile portfolios. This gives us  $2 (\beta) \times 10 (\rho^-)$  portfolios, making a total of twenty downside correlation portfolios. MKT is the CRSP value-weighted returns of all stocks. SMB and HML are the size and the book-to-market factors (constructed by Fama and French (1993)), WML is the return on the zero-cost strategy of going long past winners and shorting past losers (constructed following Carhart (1997)). T-statistics are computed using Shanken (1992) adjusted standard errors. GRS denotes the F-test of Gibbons, Ross and Shanken (1989) testing the hypothesis that the  $\alpha$ 's of all 20 portfolios are jointly zero. T-statistics that are significant at the 5% (1%) level are denoted with \* (\*\*). The sample period is from January 1964 to December 1999.

Table 4: Subsample Analysis of  $\rho^-$  Portfolios High - Low  $\alpha$ 's

	FF		FF +	WML
	10-1	t-stat	10-1	t-stat
Full Sample	0.56	4.73	0.44	3.45
Two Subsamples Jan 1964 - Dec 1981 Jan 1982 - Dec 1999	0.44 0.66	2.66 3.97	0.32 0.52	1.84 2.90
NBER Business Cycles Expansions Recessions	0.44 1.13	3.59 3.55	0.28 1.14	2.10 3.65

We report the difference in monthly  $\alpha$ 's for the tenth and the first decile beta-controlled  $\rho^-$  portfolios, with t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 3 lags. FF denotes the Fama and French (1993) model, and FF+WML denotes the Fama-French model augmented with Carhart (1997)'s WML momentum factor.

Table 5: Downside Correlation Controlling for Past Returns and Liquidity

Panel A: Downside Correlation Portfolios Controlling for Past Returns

	a	b	s	h	t(a)	t(b)	t(s)	t(h)	$\mathbb{R}^2$
1 Low $\rho^-$	-0.40	0.80	0.69	0.41	-5.00	35.02	18.30	11.58	0.88
2	-0.40	0.90	0.60	0.34	-5.19	36.09	15.72	7.22	0.91
3	-0.27	0.99	0.46	0.23	-3.79	42.29	12.37	4.67	0.93
4	-0.17	1.07	0.32	0.12	-2.49	50.12	8.80	2.60	0.94
5 High $\rho^-$	-0.07	1.10	0.11	-0.10	-1.09	59.96	3.77	-2.93	0.95

 $a_5$ - $a_1 = 0.33$  t-stat = 3.03

Panel B: Downside Correlation Portfolios Controlling for Liquidity

	a	b	s	h	t(a)	t(b)	t(s)	t(h)	$R^2$
1 Low $\rho^-$	-0.18	0.81	0.54	0.45	-2.38	30.89	12.64	12.69	0.86
2	-0.17	0.91	0.44	0.37	-2.14	36.51	11.32	7.29	0.91
3	-0.12	0.98	0.26	0.23	-1.67	46.77	8.00	4.90	0.93
4	-0.05	1.02	0.11	0.10	-0.76	60.21	3.93	2.68	0.96
5 High $\rho^-$	0.08	1.07	-0.13	-0.13	1.80	90.83	-7.70	-4.70	0.98

 $a_5$ - $a_1 = 0.26$  t-stat = 2.62

This table shows the time-series regressions of excess returns on portfolios formed by a double sort that controls for past returns and a double sort that controls for liquidity. In Panel A, we first sort stocks into quintiles by their past 6 months returns (momentum). Then, we sort stocks within each past return quintiles into additional quintiles according to  $\rho^-$ . The intersection of these quintiles forms 25 portfolios. The portfolios in Panel A are the averages across past return quintiles of the portfolios sorted by  $\rho^-$ . In Panel B, we sort stocks into quintiles by  $\beta^L$  and independently sort stocks into separate quintiles according to  $\rho^-$ . The intersection of these quintiles forms 25 portfolios. The portfolios in Panel B are the averages across  $\beta^L$  quintiles of the portfolios sorted by  $\rho^-$ . In both Panel A and Panel B, we report the coefficients from a time-series regression of the portfolio returns onto the Fama-French (1993) factors:  $r_{it} = a_i + b_i M K T_t + s_i S M B_t + h_i H M L_t + \epsilon_{it}$ . Columns labeled t() show the t-statistics of the regression coefficients computed using Newey-West (1987) heteroskedastic-robust standard errors with 3 lags. The regression  $R^2$  is adjusted for the number of degrees of freedom. January 1968 to December 1999.  $a_5$ - $a_1$  is the difference in the alphas a between the 5th quintile and the first quintile.

Table 6: Summary Statistics of the Factors

**Panel A: Summary Statistics** 

Factor	Mean	Std	Skew	Kurt	Auto
MKT	0.55*	4.40	-0.51	5.50	0.06
SMB	0.19	2.93	0.17	3.84	0.17
HML	0.32*	2.65	-0.12	3.93	0.20
WML	0.90**	3.88	-1.05	7.08	0.00
SKS	0.10	2.26	0.69	7.45	0.08
CMC	0.23*	2.06	0.04	5.41	0.10

**Panel B: Correlation Matrix** 

	MKT	SMB	HML	WML	SKS	CMC
MKT	1.00					
SMB	0.32	1.00				
HML	-0.40	-0.16	1.00			
WML	0.00	-0.27	-0.14	1.00		
SKS	0.13	0.08	0.03	-0.01	1.00	
CMC	-0.16	-0.64	-0.17	0.35	-0.03	1.00

Panel C: Regression of CMC onto Various Factors

	Constant	MKT	SMB	HML	WML
coef	0.33	-0.03	-0.44	-0.21	0.07
t-stat	4.02**	-1.47	-11.02**	-5.84**	2.65**

Panel A shows the summary statistics of the factors. MKT is the CRSP value-weighted returns of all stocks. SMB and HML are the size and the book-to-market factors (constructed by Fama and French (1993)), WML is the return on the zero-cost strategy of going long past winners and shorting past losers (constructed following Carhart (1997)), and SKS is the return on going long stocks with the most negative past coskewness and shorting stocks with the most positive past coskewness (constructed following Harvey and Siddique (2000)). CMC is the return on a portfolio going long stocks with the highest past downside correlation and shorting stocks with the lowest past downside correlation. The first two columns show the means and the standard deviations of the factors, expressed as monthly percentages. Skew and Kurt are the skewness and kurtosis of the portfolio returns. Auto refers to first-order autocorrelation. Factors with statistically significant means at the 5% (1%) level are denoted with \* (\*\*), using heteroskedastic-robust Newey-West (1987) standard errors with 3 lags. The correlation matrix between the factors is reported in Panel B. Panel C reports the regression of CMC onto MKT, SMB, HML and WML factors, with t-statistics computed using 3 Newey-West lags. T-statistics that are significant at the 5% (1%) level are denoted with \* (\*\*). The sample period is from January 1964 to December 1999.

Table 7: Pricing the Downside Correlation Portfolios (2)

#### Factor Premiums $\lambda$

	$\lambda_0$	MKT	SMB	HML	WML	CMC	GRS Test		
Model A: MKT and CMC									
Premium $(\lambda)$ t-stat	0.62 4.08**	-0.04 -0.15				0.23 2.18*	1.28 p-val=0.18		
Model B: Fama-French Factors and CMC									
Premium $(\lambda)$ t-stat	0.57 2.55*	0.03 0.09	-0.36 -1.08	0.02 0.07		0.22 2.07*	1.65 p-val=0.04*		
Model C: Fan	na-Frenc	h Factor	s, WMI	L, and C	MC				
Premium $(\lambda)$ t-stat	0.50 2.11*	0.10 0.29	-0.39 -1.15	0.11 0.33	0.64 0.88	0.21 2.01 *	1.34 p-val=0.15		

The table shows the results from Fama-MacBeth (1973) regression tests on 20 downside correlation portfolios. These portfolios are formed in the following fashion. Stocks are first sorted into two groups according to their past beta over the past year, using daily returns (high beta versus low beta). Each group consists of one half of all firms. Then, within each beta group, we rank stocks based on their  $\rho^-$ , also computed using daily data over the past year into decile portfolios. This gives us  $2 (\beta) \times 10 (\rho^-)$  portfolios, making a total of twenty downside correlation portfolios. MKT is the CRSP value-weighted returns of all stocks. SMB and HML are the size and the book-to-market factors (constructed by Fama and French (1993)), WML is the return on the zero-cost strategy of going long past winners and shorting past losers (constructed following Carhart (1997)). CMC is the return on a portfolio going long stocks with the highest past downside correlations and shorting stocks with the lowest past downside correlations. T-statistics are computed using Shanken (1992) adjusted standard errors. GRS denotes the F-test of Gibbons, Ross and Shanken (1989) testing the hypothesis that the  $\alpha$ 's of all 20 portfolios are jointly zero. T-statistics that are significant at the 5% (1%) level are denoted with \* (\*\*). The sample period is from January 1964 to December 1999.

Table 8: Macroeconomic Variables and CMC

**Panel A:** 
$$CMC_t = a + \sum_{i=1}^{3} b_i MACRO_{t-i} + \sum_{i=1}^{3} c_i CMC_{t-i} + \epsilon_t$$

		$MACRO_{t-1}$	$MACRO_{t-2}$	$MACRO_{t-3}$	Joint Sig
LEI	coef	-0.27	0.22	0.06	0.09
	t-stat	-2.27*	1.24	0.60	
HELP	coef	-0.00	-0.04	0.05	0.24
	t-stat	-0.12	-1.26	1.97	
IP	coef	-0.13	0.18	-0.02	0.16
	t-stat	-1.39	1.43	-0.22	
CPI	coef	0.17	-0.03	-0.18	0.64
	t-stat	0.43	-0.05	-0.46	
FED	coef	0.22	-0.19	-0.02	0.48
	t-stat	1.47	-0.83	-0.12	
<b>TERM</b>	coef	0.10	-0.39	0.26	0.64
	t-stat	0.46	-1.11	1.10	

Panel B:  $MACRO_t = a + \sum_{i=1}^3 b_i CMC_{t-i} + \sum_{i=1}^3 c_i MACRO_{t-i} + \epsilon_t$ 

		$CMC_{t-1}$	$CMC_{t-2}$	$CMC_{t-3}$	Joint Sig
LEI	coef	-0.02	0.02	0.01	0.62
	t-stat	-1.04	0.73	0.31	
HELP	coef	-0.48	0.03	0.18	0.00 **
	t-stat	-5.34**	0.42	1.77	
CPI	coef	-0.01	0.00	-0.01	0.51
	t-stat	-0.84	-0.17	-1.14	
IP	coef	-0.04	-0.06	0.00	0.03*
	t-stat	-1.77	-2.04*	0.03	
FED	coef	0.00	-0.03	-0.03	0.01 **
	t-stat	0.30	-1.37	-2.42*	
<b>TERM</b>	coef	-0.02	0.02	-0.01	0.03 *
	t-stat	-2.21*	1.42	-0.82	

This table shows the results of the regressions between CMC and the macroeconomic variables. Panel A lists the results from the regressions of CMC on lagged CMC and lagged macroeconomic variables, but reports only the coefficients on lagged macro variables. Panel B lists the results from the regressions of macrovariables on lagged CMC and lagged macroeconomic variables, but reports only the coefficients on lagged CMC. LEI is the growth rate of the index of leading economic indicators, HELP is the growth rate in the index of Help Wanted Advertising in Newspapers, IP is the growth rate of industrial production, CPI is the growth rate of Consumer Price Index, FED is the federal discount rate and TERM is the yield spread between 10 year bond and 3 month T-bill. All growth rates (including inflation) are computed as the differences in logs of the index at time t and time t-12, where t is in months. FED is the federal funds rate and TERM is the yield spread between the 10 year government bond yield and the 3-month T-bill yield. All variables are expressed as percentages. T-statistics are computed using Newey-West heteroskedastic-robust standard errors with 3 lags, and are listed below each estimate. Joint Sig in Panel A denotes to the p-value of the joint significance test on the coefficients on lagged macro variables. Joint Sig in Panel B denotes the p-value of the joint significance test on the coefficients of lagged CMC. T-statistics that are significant at the 5% (1%) level are denoted with \* (\*\*). P-values of less than 5% (1%) are denoted with \* (\*\*). The sample period is from January 1964 to December 1999.

Table 9: Momentum, Downside Correlation and WML Returns

Panel A: Momentum Portfolio  $\alpha$ 's from the Fama-French Model

	K=3		K	=6	K	=9	K=12			
	1 L	10 W								
Full Sa	-									
$\alpha$ t-stat	-0.54 -2.74	0.44 2.61	-0.64 -3.65	0.46 2.88	-0.67 -4.47	0.47 3.12	-0.62 -4.53	0.41 2.86		
t-stat	-2.74	2.01	-3.03	2.66	-4.47	3.12	-4.33	2.00		
Conditioning on MKT < mean - 2SE										
$\alpha$	-0.69	-1.36	-0.76	-1.32	-0.73	-1.16	-0.60	-1.35		
t-stat	-0.84	-1.52	-1.06	-1.66	-1.23	-1.64	-1.16	-2.41		

Panel B: Decile  $\rho^-$  Portfolio  $\alpha$ 's from the Fama-French Model

	Decile									
	1	2	3	4	5	6	7	8	9	10
Full Sa α t-stat	ample -0.37 -3.35	-0.30 -3.13	-0.31 -3.56	-0.24 -2.56	-0.19 -2.35	-0.16 -2.07	-0.15 -2.12	-0.10 -1.47	0.02 0.33	0.16 2.80
Condit $\alpha$ t-stat	tioning o 2.33 2.10	on MKT 1.79 1.84	< mean 1.19 0.98	- 2SE 1.02 0.74	0.15 0.14	-0.03 -0.04	0.89 1.34	0.63 1.22	0.12 0.36	-0.39 -0.73

Panel C: Regression of WML onto Various Factors

		Constant	MKT	SMB	HML	CMC	$\mathrm{Adj}\ R^2$
Model A:	coef	0.72	0.05			0.68	0.12
	t-stat	4.19**	0.73			4.82**	
Model B:		1.05		-0.41			0.10
	t-stat	6.25**	0.33	-3.20**	-2.19*		
Model C:	coef	0.86	0.04	-0.19	-0.15	0.47	0.13
	t-stat	4.87**	0.55	-1.18	-1.27	2.81 **	

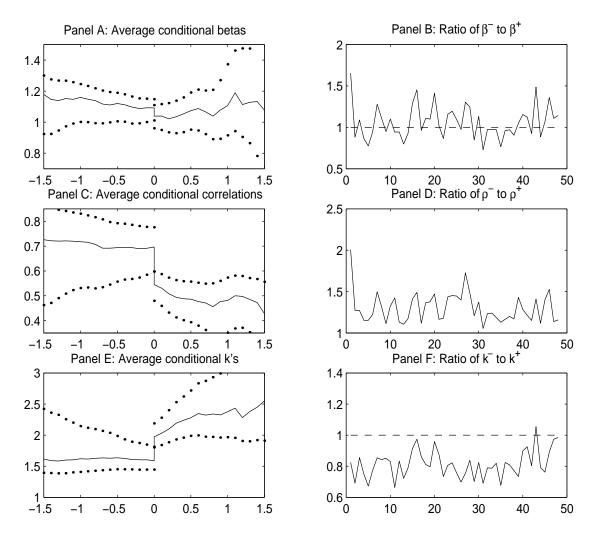
In Panels A and B, the table reports  $\alpha$ 's from a Fama-French (1993) model for the 40 momentum portfolios and the decile  $\rho^-$  portfolios. The 40 momentum portfolios are formed using a J=6 month formation period, with holding periods of K=3, 6, 9 or 12 months, with 10 deciles within in each K. The decile  $\rho^-$  portfolios are the same portfolios in Table (2). Alpha's from two samples are reported: over the full sample, and over a sample conditioned on the market return (MKT) being below two standard deviations from its mean. The full sample period is from January 1964 to December 1999. There are 41 observations where the market is less than two standard deviations from its unconditional mean, where both the mean and standard deviation are computed using the full sample. In Panel C, the table reports the time-series regression of the momentum factor, WML, onto various other factors. MKT is the market, SMB and HML are Fama-French (1993) factors, CMC is the downside risk factor, and SKS is the Harvey-Siddique (2000) skewness factor. The t-stat is computed using Newey-West (1987) heteroskedastic-robust standard errors with 3 lags. In Panel C, t-statistics that are significant at the 5% (1%) level are denoted with \* (\*\*). The sample period is from January 1964 to December 1999.

Table 10: Fama-MacBeth Regression Tests of the Momentum Portfolios Factor Premiums  $\lambda$ 

	$\lambda_0$	MKT	SMB	HML	CMC	WML	Joint Sig			
Model A: Fama-French Model										
Premium $(\lambda)$ t-stat		2.04 1.66		-0.98 -2.17*			p-val=0.07			
Model B: Usi	ng MKT	and CN	<b>И</b> С							
Premium $(\lambda)$ t-stat		1.98 2.32*			0.73 2.38*		p-val=0.03 *			
Model C: Far	Model C: Fama-French Factors and CMC									
Premium $(\lambda)$ t-stat		1.73 1.22			1.02 2.43*		p-val=0.00 **			
Model D: Fama-French Factors, CMC and WML										
Premium $(\lambda)$ t-stat	-0.24 -0.19	0.45 0.29	0.50 1.04	0.64 1.08	0.98 2.01*	0.84 2.74**	p-val=0.01 **			

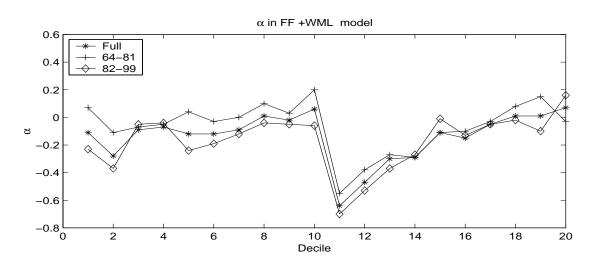
This table shows the results from the Fama-MacBeth (1973) regression tests on the 40 momentum portfolios sorted by past 6 months returns. MKT, SMB and HML are Fama and French (1993)'s three factors and CMC is the downside risk factor. WML is return on the zero-cost strategy going long past winners and shorting past losers (constructed following Carhart (1997)). In the first stage we estimate the factor loadings over the whole sample. The factor premia,  $\lambda$ , are estimated in the second-stage cross-sectional regressions. The last column of the table reports p-values from  $\chi^2$  tests on the joint significance of the betas of each model. All statistics are computed using Shanken (1992) standard errors. T-statistics that are significant at the 5% (1%) level are denoted with \* (\*\*). The sample period is from January 1964 to December 1999.

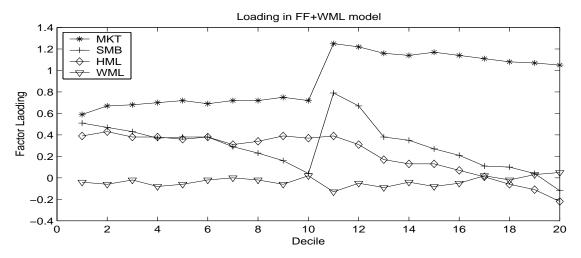
Figure 1: Downside and Upside Moments of Industry Portfolios



The left column of the figure shows upside and downside betas in Panel A, upside and downside correlations in Panel C and the ratio of upside and downside total portfolio volatility to market volatility in Panel E. All of these are averaged across the 48 Fama and French (1997) industry portfolios. The graph in Panel A is constructed as follows (Panels C and E are similar). The figure displays the average  $\beta^-(\theta)$  across the portfolios, where  $\theta$  is x standard deviations below the unconditional mean of the market. For example, at x=-1, the figure plots the average industry  $\beta^-(\theta=\overline{MKT}-SE_{MKT})$ , where  $\overline{MKT}$  is the unconditional mean of the excess market return and  $SE_{MKT}$  is the unconditional volatility of the excess market return. At x=0, the figure plots the average  $\beta^-(\theta=\overline{MKT})$ . Similarly, on the RHS of the x-axis for  $x\geq 0$ , the figure displays the average  $\beta^+(\theta)$ , for  $\theta$  representing x standard deviations above the mean of the market. There are two points plotted at x=0 representing the average  $\beta^-(\theta=\overline{MKT})\equiv\beta^-$  and the average  $\beta^+(\theta=\overline{MKT})\equiv\beta^+$ . The 95% standard error bounds shown in dotted lines in Panels A, C, and E are produced by bootstrap with 10,000 simulations. Panels B, D and F, in the right column of the figure, report the ratio of  $\beta^-/\beta^+$ , (Panel B), the ratio of  $\rho^-/\rho^+$  (Panel D) and the ratio of  $k^-/k^+$  (Panel E) for each of the 48 industry portfolios (numbered 1-48 on the x-axis). All of these ratios are computed at the conditioning level of  $\theta=\overline{MKT}$ . Data is sampled monthly from Jan 1964 - Dec 1999.

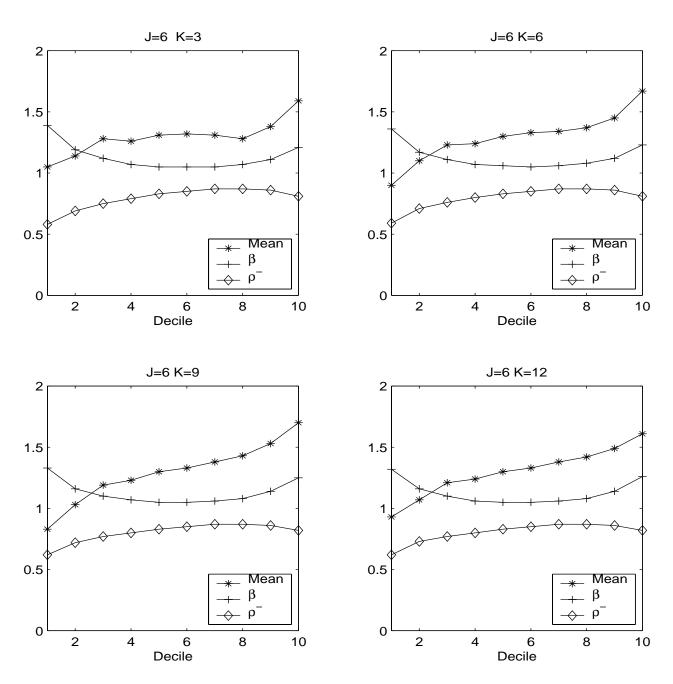
Figure 2: Alphas and Factor Loadings of the Downside Correlation Portfolios





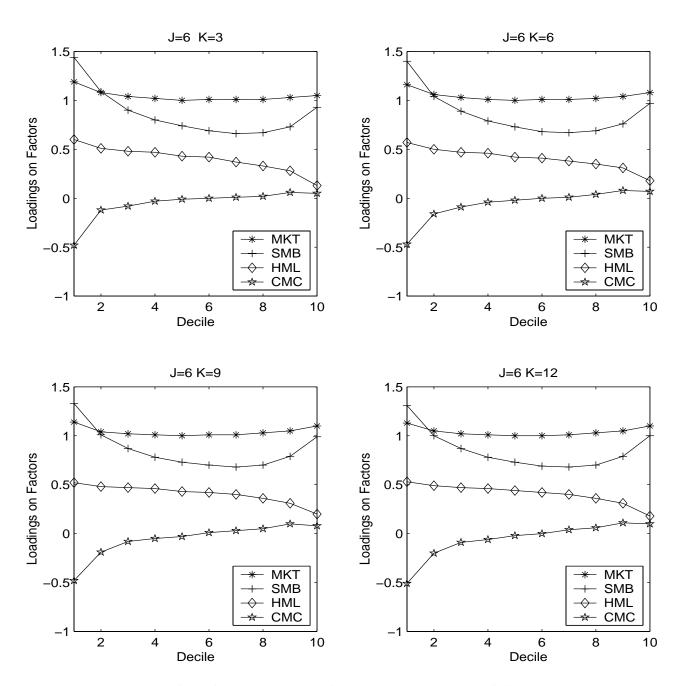
The top panel shows portfolio alphas from the  $20~\rho^-$  portfolios. These portfolios are formed in the following fashion. First, stocks are sorted into two groups according to their past beta over the past year, using daily returns (high beta versus low beta). Each group consists of one half of all firms. Then, within each beta group, we rank stocks based on their  $\rho^-$ , also computed using daily data over the past year into decile portfolios. This gives us  $2~(\beta) \times 10~(\rho^-)$  portfolios, making a total of twenty downside correlation portfolios. The portfolios 1-10 (11-20) are from the low (high) beta group. The  $\alpha$ 's are from a model of the Fama-French (1993) factors, augmented with Carhart (1997)'s WML momentum factor and are shown over three periods: over the full sample, from Jan 1964 - Dec 1981, and from Jan 1982 - Dec 1999. The bottom panel shows the portfolio factor loadings on the MKT, SMB, HML and WML factors over the full sample.

Figure 3: Average Return,  $\beta$ ,  $\rho^-$  of Momentum Portfolios



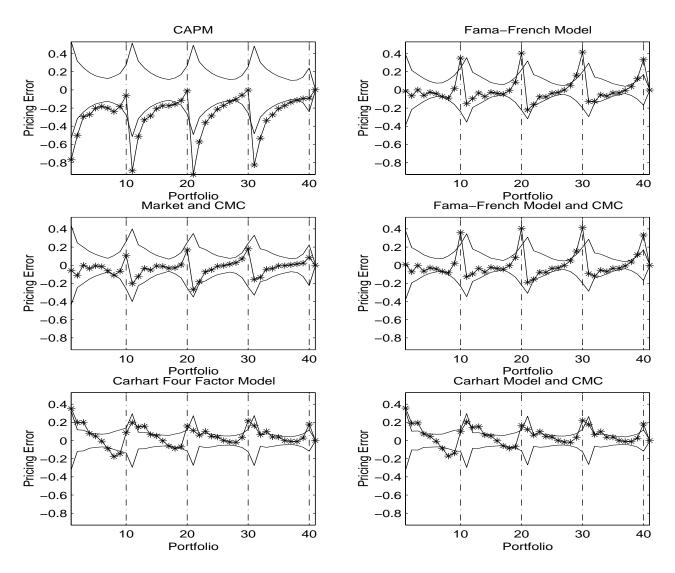
These plots show the average monthly percentage returns,  $\beta$  and  $\rho^-$  of the Jegadeesh and Titman (1993) momentum portfolios. J refers to formation period and K refers to holding periods. For each month, we sort all NYSE and AMEX stocks into decile portfolios based on their returns over the past J=6 months. We consider holding periods over the next 3, 6, 9 and 12 months. This procedure yields 4 strategies and 40 portfolios in total. The sample period is from January 1964 to December 1999.

Figure 4: Loadings of Momentum Portfolios on Factors



These plots show the loadings of the Jegadeesh and Titman (1993) momentum portfolios on MKT, SMB, HML and CMC. Factor loadings are estimated in the first step of the Fama-MacBeth (1973) procedure (equation (B-2)). J refers to formation period and K refers to holding periods. For each month, we sort all NYSE and AMEX stocks into decile portfolios based on their returns over the past J=6 months. We consider holding periods over the next 3, 6, 9 and 12 months. This procedure yields 4 strategies and 40 portfolios in total. MKT, SMB and HML are Fama and French (1993)'s three factors and CMC is the downside correlation risk factor. The sample period is from January 1964 to December 1999.

Figure 5: Pricing Errors of GMM Estimation (HJ method)



These plots show the pricing errors of various models considered in Section 5.2. Each star in the graph represents one of the 40 momentum portfolios with J=6 or the risk-free asset. The first ten portfolios correspond to the K=3 month holding period, the second ten to the K=6 month holding period, the third ten to the K=9 month holding period, and finally the fourth ten to the K=12 holding period. The 41st asset is the risk-free asset. The graphs show the average pricing errors with asterixes, with two standard error bands in solid lines. The units on the y-axis are in percentage terms. Pricing errors are estimated following computation of the Hansen-Jagannathan (1997) distance. The Carhart (1997) four-factor model consists of MKT, SMB, HML and WML factors.