

**RESEARCH**

# Residual Volatility and Average Returns

December 2012

**Wes Crill**  
RESEARCH  
Analyst

*Wes earned a BS in materials science engineering from North Carolina State University before pursuing a PhD in the same program.*

**Ronnie R. Shah**  
RESEARCH  
Senior Associate

*Ronnie completed his PhD in finance at the University of Texas and holds an MSc in finance from the London School of Economics, as well as a BS in economics from the University of Pennsylvania Wharton School.*

Previous academic research shows stocks with low residual volatility have outperformed stocks with high residual volatility. This paper examines whether residual volatility can be used to improve real-world investment strategies, and finds that incorporating residual volatility is unlikely to improve performance for an investment process that accounts for momentum and small growth.

**INTRODUCTION**

A number of academic papers, starting with Ang et al. (2006), document a negative relation between average returns and residual volatility measured using daily returns from the previous month.<sup>1</sup> Residual volatility is volatility that is unexplained by a factor model.

Financial theory suggests residual volatility should not be related to expected returns due to diversification. A robust portfolio construction process reduces volatility by investing in a large number of assets that span different geographies and industries. Firm-specific volatility largely washes out. What is left is systematic volatility related to common sources of return variation.

---

The helpful comments of Jim Davis, Ken French, Marlana Lee, Gerard O'Reilly, and Sunil Wahal are gratefully acknowledged.

1. Academic papers often refer to residual volatility as idiosyncratic volatility.

The material in this publication is provided solely as background information for registered investment advisors and institutional investors and is not intended for public use.

Despite these explanations of why residual volatility should not affect future returns, several studies have found reliable in-sample return predictability. In this study, we take a deeper look at strategies based on residual volatility by addressing the following questions:

- Do portfolios that overweight low residual volatility stocks outperform the market?
- Can a long/short portfolio that is long low residual volatility and short high residual volatility stocks earn abnormal profits?
- Can we enhance a long-only equity portfolio strategy by excluding high residual volatility stocks?
- How is residual volatility related to size, value, and momentum?
  - How does the relation between residual volatility and average returns differ across the size spectrum?
  - Does negative momentum or the well-documented poor performance of small growth explain the residual volatility phenomenon?

Consistent with prior research, we find that high residual volatility stocks have lower average returns than low residual volatility stocks. Our findings, however, cast doubt on whether residual volatility can be used to improve a robust investment process by considering different portfolio implementations.

- **Long-only.** High residual volatility stocks have abysmally poor returns, but there is very little difference in returns between low and medium residual volatility stocks. Thus, long-only strategies that tilt toward low volatility stocks do not outperform the market in-sample.
- **Long/short.** Many high residual volatility stocks are very small, and as a group represent 2.4% of the aggregate market cap. Long/short portfolios that short high residual volatility stocks are unlikely to generate abnormal profits due to the high costs of shorting small stocks and the high turnover of these portfolios.
- **Portfolio enhancement.** Research has shown that stocks that have negative past momentum or relatively low recent returns (Jegadeesh and Titman 1993), and small firms with low book-to-market ratios (Fama and French 2008)

have poor historic returns. Accounting for these effects by excluding negative momentum and small growth stocks substantially reduces the average return differential between low and high volatility portfolios. Exclusion rules based on residual volatility are unlikely to improve performance of a long-only portfolio that already monitors momentum and small growth.

#### PAST LITERATURE ON RESIDUAL VOLATILITY

To begin our discussion on residual volatility, we rely heavily on past academic studies. Ang et al. (2006) find that high residual volatility stocks underperform low residual volatility stocks. They define residual volatility as the standard deviation of residuals from a regression of individual daily stock returns on the Fama/French three factors. These authors assert that their findings cannot be explained by existing asset pricing models. In a follow-up paper, Ang et al. (2009) show that this result extends to twenty-three developed international markets.

Some papers in the academic literature are critical of the ability of daily measures of residual volatility to predict returns. Han and Lesmond (2011) suggest that the residual volatility effect is explained by two market microstructure effects: zero returns and bid-ask bounce. On days when no trades for a particular stock occurred, the stock's return was recorded as zero. Zero return days reduce volatility estimates. Bid-ask bounce increases volatility, as daily returns are measured using closing prices instead of the closing bid-ask spread midpoint. When trades for a security on consecutive days occur at different points in the bid-ask spread, the security will have a non-zero return even if the true price didn't change. Stocks with large bid-ask spreads therefore tend to mechanically exhibit greater volatility when returns are measured using closing prices. Han and Lesmond (2011) control for these measurement biases by estimating residual volatility using closing bid-ask midpoint prices (instead of closing prices) and removing zero returns. Their analysis shows these controls substantially reduce the profitability of residual volatility strategies.

Huang et al. (2011) and Fu (2009) propose that the residual volatility anomaly is explained by one-month reversals.<sup>2</sup> Pontiff (2006) hypothesizes that residual volatility effects exist in equilibrium due to limits of arbitrage. In his reasoning, traders are unable to capitalize on the return phenomenon due to the costliness of trading high residual volatility stocks. Duan, Hu, and McLean (2010) take the analysis one step further by showing that firms that have high short interest (and are costly to short as fewer lendable shares are available to borrow) exhibit substantially lower returns when residual volatility is high.

#### DATA AND METHODOLOGY

The starting sample for this study includes all NYSE, AMEX, and NASDAQ stocks listed on the Center for Research in Security Prices (CRSP) return files during the period January 1963–December 2011. We exclude stocks with beginning-of-month prices of less than \$2 to ensure our results are not driven by extremely low-priced stocks.<sup>3</sup> To mitigate the effects of delisting returns, we adjust returns for delisted firms according to the methodology outlined in Shumway (1997). Information on stock returns, stock prices, and shares outstanding is taken from the CRSP database. Factor returns and percentile breakpoints for portfolios formed on market capitalization, book-to-market, and momentum are provided by Ken French.<sup>4</sup>

The bulk of our empirical results rely on three measures. The first is firm size, which is equal to the beginning-of-the-month market capitalization, calculated as the share price multiplied by the number of shares outstanding. For firms with multiple share classes, we take the sum of market capitalization across all share classes as our estimate of firm size.

Second, as a proxy for the trading costs, we take the latest, non-zero bid-ask spread from closing monthly prices. Reliable bid-ask spread data begins in 1993. Bid-ask spreads are calculated as:

$$\text{Spread} = (\text{Ask} - \text{Bid}) / \text{Ask}$$

Third, residual volatility is estimated from a regression of prior excess stock returns  $r$  on the excess market return  $Rm-r_f$ , the small-minus-big portfolio (SMB) and high-minus-low portfolio (HML). This methodology for calculating residual volatility is borrowed from Ang et al. (2006). The three explanatory variables on the right-hand side of the regression are the components used in the Fama/French three-factor model.<sup>5</sup>

$$r = \alpha + b [Rm-r_f] + s [SMB] + h [HML] + \epsilon$$

The majority of academic studies pay particular attention to  $\alpha$ , the alpha or abnormal return, and the factor loadings or  $b$ ,  $s$ , and  $h$ . For this project, we are interested in the error term,  $\epsilon$ , which represents the residual or unexplained return after accounting for risk factors. The final measure of residual volatility is the standard deviation of the error term  $\epsilon$ .

2. One-month reversals are the tendency for underperforming stocks in a given month to outperform in the following month. (See, for example, Jegadeesh 1990.)

3. Including stocks with prices less than \$2 does not materially affect the results.

4. For more information, see <http://www.dartmouth.edu/~kfrench/>.

5. See Fama and French (1993) for more information on the Fama/French three-factor model.

**DO RESIDUAL VOLATILITY PORTFOLIO STRATEGIES EARN HIGHER ABNORMAL RETURNS?**

The first step toward understanding residual volatility strategies involves examining portfolio returns of stocks sorted on volatility. The first column of Table 1 reports value-weighted returns and the percentage of total market capitalization for the five residual volatility portfolios. The last five columns of Table 1 sort stocks into five portfolios based on market capitalization (horizontal axis on the top). Those stocks are then subdivided into five portfolios based on residual volatility (vertical axis on the left) for a total of twenty-five portfolios.

The left-most column in Table 1 shows that the high residual volatility portfolio at the bottom of the first column, despite containing 20% of the total number of stocks, represents 2.38% of the aggregate market. This portfolio contains many small firms. Returns for this portfolio are close to zero, while returns for the other four volatility buckets range from 0.75% to 1.02%. Over the period 1963–2011, a long-only manager who tilted toward low volatility stocks by buying the lowest 20% of stocks based on volatility would generate a monthly return of 0.87%, compared to a return of 0.86% for the market. Since high residual volatility stocks represent a small segment of the market, they have very little impact on the market return.

Table 1. **DEPENDENT RETURN SORTS ON SIZE AND RESIDUAL VOLATILITY**

Residual volatility is estimated for each stock using the volatility of the residual from a daily Fama/French three-factor model using data from previous two months. The first column contains the returns from a univariate sort on residual volatility. The last five columns contain bivariate-sorted portfolios for which stocks are first sorted into five groups using NYSE size breakpoints and then into five groups based on residual volatility. Value-weighted returns are reported for each portfolio the month following formation. The label “Number of Firms” refers to the average number of firms over time in each size/residual volatility portfolio. The average percentage of total market capitalization for each portfolio is given in brackets. High–Low denote returns from long-short portfolios that go long the highest residual volatility portfolio and short the lowest residual volatility portfolio for a given size quintile. T-stat indicates robustness. Monthly turnover for the lowest and highest volatility-sorted portfolios is reported.

	ALL	SMALL	ME-2	ME-3	ME-4	LARGE
<b>Low Volatility</b>	0.87% [57.51%]	1.36% [0.67%]	1.25% [0.81%]	1.07% [1.24%]	1.02% [2.39%]	0.83% [21.11%]
<b>Vol-2</b>	0.95% [23.75%]	1.49% [0.68%]	1.38% [0.80%]	1.27% [1.23%]	1.10% [2.43%]	0.83% [17.82%]
<b>Vol-3</b>	1.02% [10.89%]	1.34% [0.62%]	1.28% [0.78%]	1.24% [1.21%]	1.14% [2.41%]	0.83% [15.10%]
<b>Vol-4</b>	0.75% [5.47%]	0.92% [0.54%]	1.13% [0.77%]	1.13% [1.19%]	1.13% [2.38%]	0.79% [12.02%]
<b>High Volatility</b>	0.04% [2.38%]	-0.15% [0.40%]	0.36% [0.75%]	0.52% [1.16%]	0.66% [2.30%]	0.70% [9.20%]
<b>NUMBER OF FIRMS*</b>	856	494	136	91	72	62
<b>High–Low Return</b>	-0.84%	-1.51%	-0.90%	-0.56%	-0.36%	-0.13%
<b>Stdev</b>	7.08%	6.21%	6.92%	6.71%	6.56%	5.15%
<b>t-stat</b>	(-2.80)	(-5.79)	(-3.08)	(-1.97)	(-1.31)	(-0.60)
<b>Low (Buy) T/O</b>		31.5%	34.9%	34.9%	37.0%	28.8%
<b>High (Sell) T/O</b>		42.6%	45.9%	44.0%	41.5%	38.7%

\* Totals may not add due to rounding.

The five right-most columns in Table 1 sort stocks on size and then residual volatility. Returns on the first four quintiles based on residual volatility (shown in the first four rows) average 0.79% to 1.49% per month, while the highest residual volatility group (in the last row) has returns that average -0.15% to 0.70% per month. The variation in returns due to residual volatility within a size group is best observed in the long-short portfolios (“High–Low Return”), which are long the highest residual volatility stocks and short the lowest residual volatility stocks. As with the first column, all five long-short returns are negative. However, the t-statistic for the long-short return is only greater than 2 for the two smallest size groups, suggesting the relation between residual volatility and average returns is concentrated among small stocks.

The average returns reported in Table 1 assume zero transaction costs. A portfolio that buys low volatility stocks and sells high volatility stocks incurs trading costs that are dependent on the level of turnover and the bid-ask spread of stocks that are purchased or sold. The last two rows report buy and sell monthly turnover for each of the five residual volatility long/short portfolios. The turnover for each portfolio ranges from 28.8% to 45.9%.

Small firms are more costly to trade due to higher bid-ask spreads. Table 2 reports median bid-ask spreads for portfolios sorted on residual volatility and size.<sup>6</sup> Bid-ask spreads decrease from small to big and increase from low to high volatility within each size group. Estimating transaction costs on long/short portfolios is difficult due to lack of information on shorting costs. When a long/short manager initiates a short position, he typically pays a commission that is similar to one paid when purchasing a stock and pays a borrowing cost that can exceed 10% per year for certain small securities. Shorting securities is generally more costly than purchasing securities.

The long/short returns reported in Table 1 are probably not achievable due to the high costs of trading small stocks. As a crude estimate, we calculate the transaction costs of the smallest long/short portfolio in Table 1 by multiplying the turnover of the long and short portfolios with the respective bid-ask spread estimates in Table 2.

$$T\text{-cost: } 31.5\% \times 1.63\% + 42.6\% \times 2.33\% = 1.51\% \text{ per month}$$

After accounting for transaction costs, the alpha of the smallest long/short portfolio is close to zero.

Table 2. **BID-ASK SPREADS FOR PORTFOLIOS DEPENDENTLY SORTED ON SIZE AND RESIDUAL VOLATILITY**

Residual volatility is estimated for each stock using the volatility of the residual from a daily Fama/French three-factor model using data from previous two months. The median bid-ask spread in percent for each portfolio is calculated as  $Ask/Bid - 1$ .

	SMALL	ME-2	ME-3	ME-4	LARGE
<b>Low Volatility</b>	1.63%	0.84%	0.65%	0.53%	0.44%
<b>Vol-2</b>	1.89%	0.98%	0.74%	0.59%	0.45%
<b>Vol-3</b>	2.00%	1.00%	0.75%	0.61%	0.47%
<b>Vol-4</b>	2.13%	0.96%	0.67%	0.57%	0.44%
<b>High Volatility</b>	2.33%	0.95%	0.60%	0.46%	0.35%

6. Reliable bid-ask spread data begins in 1993.

**CAN RESIDUAL VOLATILITY STRATEGIES BE USED TO ENHANCE A LONG-ONLY STRATEGY?**

The highest residual volatility portfolio has puzzlingly low average returns and represents only 2.4% of the total market. Are these firms reflecting known factor exposures? In this section, we control for two sources of underperformance: small growth firms (Fama and French 2008), and firms with negative momentum or poor recent past returns (Jegadeesh and Titman 1993). Our process involves excluding firms that either have had negative past return momentum or are characterized as small growth. Details of the exclusion rules can be found in the appendix. If the low returns on high residual volatility stocks remain after accounting for negative momentum and small growth, removing these firms may improve performance for a long-only strategy.

We exclude downward momentum and small growth firms in two different ways, which are presented in Table 3. For Panel A, stocks are first sorted into the five portfolios based on residual volatility. Stocks with downward momentum or small growth characteristics are then removed from each portfolio. The table reports quintile returns, average firm counts, and high–low return spreads. The percentages in brackets report the aggregate capitalization affected by excluding negative momentum and/or small growth stocks. More than half of the capitalization in the highest volatility portfolio is removed when screening on both negative momentum and small growth.

Table 3. RETURN SORTS AFTER PORTFOLIO EXCLUSIONS

Residual volatility is estimated using daily stock returns over the past two months. Stocks are sorted into quintiles based on residual volatility, and value-weighted average monthly returns are reported. The high–low return is the average long/short return associated with buying the highest volatility quintile of stocks and selling the lowest quintile. T-stat indicates robustness. The average percentage of total market capitalization excluded for each portfolio due to the exclusions is given in brackets. In Panel A, volatility breakpoints are determined for the full universe, and then stocks are excluded from each portfolio. The portfolios in Panel B are formed using breakpoints set after the exclusions are applied.

PANEL A: VOLATILITY BREAKPOINTS DETERMINED BEFORE EXCLUSIONS								
	RESIDUAL VOLATILITY RANKING						HIGH–LOW	
	LOW	2	3	4	HIGH	AVG # FIRMS	VW RET	T-STAT
All Stocks	0.87%	0.95%	1.02%	0.75%	0.04%	4,284	-0.84%	(-2.80)
Excluding (-) Momentum [Cap Excluded]	0.90% [24.6%]	1.04% [32.4%]	1.25% [36.1%]	1.09% [37.8%]	0.46% [38.8%]	2,745 [28.8%]	-0.44%	(-1.47)
Ex (-) Mom. & Small Growth [Cap Excluded]	0.90% [25.1%]	1.02% [34.2%]	1.24% [41.0%]	1.11% [46.9%]	0.56% [52.2%]	2,239 [30.8%]	-0.34%	(-1.11)

  

PANEL B: VOLATILITY BREAKPOINTS DETERMINED AFTER EXCLUSIONS								
	RESIDUAL VOLATILITY RANKING						HIGH–LOW	
	LOW	2	3	4	HIGH	AVG # FIRMS	VW RET	T-STAT
All Stocks	0.87%	0.95%	1.02%	0.75%	0.04%	4,284	-0.84%	(-2.80)
Excluding (-) Momentum	0.90%	1.03%	1.24%	1.16%	0.62%	2,745	-0.28%	(-0.94)
Ex (-) Mom. & Small Growth	0.88%	1.01%	1.23%	1.12%	0.81%	2,239	-0.07%	(-0.21)

Screening on downward momentum reduces the spread in monthly returns by nearly half, from -0.84% to -0.44%. The average return of the portfolio with the highest residual volatility improves from 0.04% per month to 0.46% when downward momentum stocks are excluded. The returns in the lowest volatility quintile are largely unaffected by the screen. Applying both exclusion rules explains close to 60% of the high-low return spread.

One potential issue with the first panel of Table 3 could be the lack of an equal number of firms across the different portfolios (since high residual volatility portfolios have a greater number of screened firms). We perform the test another way by first dropping negative momentum and small growth stocks, and then forming residual volatility portfolios. Ranking stocks on residual volatility after screening produces portfolios with an equal number of stocks. The results in Panel B are similar to those of Panel A. Both screens narrow the return spread between the high and low volatility quintiles, with downward momentum explaining a larger proportion of the difference. Combining both screens reduces the returns spread from -0.84% per month to -0.07%.

## CONCLUSION

The purpose of this study is to better understand the benefits and limitations of strategies based on residual volatility. Stocks with high residual volatility historically have had poor returns. Despite the attention these strategies have generated, the highest 20% of residual volatility stocks represent only 2.4% of the total market capitalization and consist of hard-to-trade small stocks. The relation between residual volatility and average returns is strongest among micro- and small-capitalization stocks. Returns on a long/short residual volatility portfolio are most likely not achievable given the large estimated trading costs. Finally, the return spread between high volatility stocks and low volatility stocks is substantially reduced when controlling for small growth and downward momentum.

## APPENDIX: DOWNWARD MOMENTUM AND SMALL GROWTH EXCLUSIONS

To formulate our exclusions, we compute book-to-market values using beginning-of-month market capitalization and the fiscal year-end book value from the prior year. Forming book values for firms requires data for stockholders' equity, deferred taxes, investment tax credit, and preferred stock value. The ratio of book value to market equity is then calculated for all firms with a valid book value for the prior year and non-zero market equity for the current month. Momentum for each stock is calculated monthly using cumulative returns over the prior year, skipping the preceding month. For month  $t$ , all stocks with a valid entry for market equity in month  $t-12$  and return in month  $t-2$  are included.

A stock is characterized as small growth if its market cap falls below the NYSE median and its book-to-market value is less than the NYSE thirtieth percentile. Stocks are considered to have downward momentum if their prior cumulative return is below the NYSE thirtieth percentile. Both classifications are determined monthly.

---

### ***Past performance is no guarantee of future results.***

*Diversification neither assures a profit nor guarantees against loss in a declining market.*

©2012 Dimensional Fund Advisors LP. All rights reserved. *Unauthorized copying, reproducing, duplicating, or transmitting of this material is prohibited. This information is for educational purposes only and should not be considered investment advice or an offer of any security for sale.*

*Dimensional Fund Advisors LP is an investment advisor registered with the Securities and Exchange Commission.*

## REFERENCES

- Amihud, Yakov. 2002. "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects." *Journal of Financial Markets* 5: 31-56.
- Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang. 2006. "The Cross-Section of Volatility and Expected Returns." *Journal of Finance* 59: 259-299.
- Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang. 2009. "High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence." *Journal of Financial Economics* 91: 1-23.
- Bali, Turan G. and Nusret Cakici. 2008. "Idiosyncratic Volatility and the Cross-Section of Expected Returns." *Journal of Financial and Quantitative Analysis* 43: 29-58.
- Campbell, John Y. 1996. "Understanding Risk and Return." *Journal of Political Economy* 104: 298-345.
- Carhart, Mark M. 1997. "On Persistence in Mutual Fund Performance." *Journal of Finance* 52: 57-82.
- Duan, Ying, Gang Hu, and R. David McLean. 2010. "Costly Arbitrage and Idiosyncratic Risk: Evidence from Short Sellers." *Journal of Financial Intermediation* 19: 564-579.
- Fama, Eugene F. and Kenneth R. French. 1993. "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics* 33: 5-56.
- Fama, Eugene F. and Kenneth R. French. 2008. "Dissecting Anomalies." *Journal of Finance* 63: 1653-1678.
- French, Kenneth R., William G. Schwert, and Robert F. Stambaugh. 1987. "Expected Stock Returns and Volatility." *Journal of Financial Economics* 19: 3-29.
- Fu, Fangjian. 2009. "Idiosyncratic Risk and the Cross-Section of Expected Stock Returns." *Journal of Financial Economics* 91: 24-37.
- Han, Yufeng and David A. Lesmond. 2011. "Liquidity Biases and the Pricing of Cross-Sectional Idiosyncratic Volatility." *Review of Financial Studies* 24: 1-40.
- Huang, Wei, Qianqiu Liu, Ghon S. Rhee, and Liang Zhang. 2009. "Return Reversals, Idiosyncratic Risk, and Expected Returns." *Review of Financial Studies* 23: 147-168.
- Jegadeesh, Narasimhan, 1990. "Evidence of Predictable Behavior of Security Returns." *Journal of Finance* 45: 881-898.
- Jegadeesh, Narasimhan, and Sheridan Titman. 1993. "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency." *Journal of Finance* 48: 65-91.
- Jiang, Xiaoquan and Bong-Soo Lee. 2006. "The Dynamic Relation between Returns and Idiosyncratic Volatility." *Financial Management* 35: 43-65.
- Merton, Robert C. 1987. "A Simple Model of Capital Market Equilibrium with Incomplete Information." *Journal of Finance* 42: 483-510.
- Pontiff, Jeffrey. 2006. "Costly Arbitrage and the Myth of Idiosyncratic Risk." *Journal of Accounting and Economics* 42: 35-52.
- Shumway, Tyler. 1997. "The Delisting Bias in CRSP Data." *Journal of Finance* 52: 327-340.